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Adaptive Block Coordinate Descent for Distortion Minimization

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Abstract

We present a new unified algorithm for optimizing geometric energies and computing positively oriented simplicial mappings. Its major improvements over the state-of-the-art are: adaptive partition of vertices into coordinate blocks with the blended local-global strategy, introduction of new distortion energies for repairing inverted and degenerated simplices, modification of standard rotation-invariant measures, introduction of displacement norm for improving convergence criteria and for controlling the proposed local-global blending. Together these improvements form the basis for Adaptive Block Coordinate Descent (ABCD) algorithm aimed at robust geometric optimization. Our algorithm achieves state-of-the-art results in distortion minimization, even with highly distorted invalid initializations that contain thousands of inverted and degenerated elements. We show over a wide range of 2D and 3D problems that ABCD is more robust than existing techniques in locally injective mappings.

CCS Concepts

• ***Theory of computation*** → *Nonconvex optimization;* • ***Computing methodologies*** → *Computer graphics;*

1. Introduction

Computing inversion-free mappings with low distortions on triangulated domains is a fundamental problem in computer graphics, geometrical modeling and physical simulations. This problem often results in non-linear optimization of geometric energies commonly expressed in a finite element manner as a weighted sum of distortion densities $\mathcal{D}$ over simplices $c \in \mathcal{C}$:

$$E\left(f[\boldsymbol{x}]\right) = \sum_{c \in \mathcal{C}} W_c \mathcal{D}(df_c)\,, \qquad (1)$$

where $\boldsymbol{x}$ is the column stack of vertex positions under piecewise affine mapping f and df_c denotes the Jacobian of f on c, modulo a rigid transformation of c from the source to the target domain.

The existing solutions to the above problem typically fall into two major categories: (1) *map fixers*: algorithms focused on the injectivity of maps ; (2) *core-solvers*: iterative descent algorithms focused on minimizing rotation-invariant energies. Map fixers, such as LBD [KABL15] and SA [FL16], are aimed at repairing foldovers of non-injective maps and restraining their geometric distortions into a finite range. Core-solvers start from a one-to-one initialization and ensure that optimization results remain one-to-one, at least locally. Among other methods, recent studies on core-solvers include: BQCN [ZBK18], AKVF [CBSS17], SLIM [RPPSH17] and CM [SPSH*17]. Efficient design of map fixers and core-solvers has been extensively studied by the computer graphics community separately. A unified framework for both approaches has received little attention in the literature, largely due to the inherent differences in the underlying structures of both approaches.

We propose the **Adaptive Block Coordinate Descent (ABCD)** algorithm that successfully tackles both problems within the same routine on triangle and tetrahedral meshes. Our algorithm achieves state-of-the-art results in distortion minimization, even under complex positional constraints and highly distorted invalid initializations that contain thousands of degenerated and inverted elements. Consequently, compared with core-solvers, our algorithm is much more robust. At the same time, compared with recent map fixer methods, our algorithm achieves superior results, making some previously-intractable problems to become applicable.

2. The method

Starting with an invalid non-injective initial map, ABCD behaves as a modified block coordinate descent until the current mapping is cleared of foldovers and singularities. Then, the algorithm converges rapidly into the chosen distortion solver. This behavior is attained mostly by redesigning distortion energies, and by employing adaptive strategy that continuously modifies coordinate blocks and the related solver parameters. The main stages of ABCD include:

1. Computing descent field $\boldsymbol{d}$ of the current energy $E[\boldsymbol{x}]$, obtained from an alternating sequence of the proposed *map fixer measure* $\mathcal{D}_{\text{fix}}$ and a modified rotation-invariant distortion $\hat{\mathcal{D}}$.
2. Estimating the vertex partitioning threshold K by using the proposed *local-global blending* strategy.
3. Vertex partitioning into coordinate blocks $\{\boldsymbol{x}_B \,|\, B \in \mathbf{B}\}$, such that $\boldsymbol{d}$ attains K-Lipschitz continuity in each block $B \in \mathbf{B}$.

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4. Parallel optimization of (1) in coordinate blocks $B \in \mathbf{B}$.
5. Checking for termination using the characteristic gradient norm [ZBK18] and the proposed *displacement norm* criteria.

If $\sigma_1, \ldots, \sigma_n$ are signed singular values of df_c for $c \in \mathcal{C}$, then

$$\mathcal{D}_{\text{fix}}\left(df_c\right) \triangleq \begin{cases} \Lambda - \prod \sigma_i & \prod \sigma_i \leq 0 \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

where Λ is the uniform penalty cost of an invalid simplex. For an efficient integration into our algorithm, a standard rotation-invariant distortion $\mathcal{D}(df_c)$ (e.g., conformal, isometric distortions) is modified to a new measure $\hat{\mathcal{D}}$ as follows: on positively oriented simplices $\hat{\mathcal{D}}(df_c)$ equals $\mathcal{D}(df_c)$ plus the barrier term to prevent inversions and collapses of c, whereas on flipped elements $\hat{\mathcal{D}}(df_c) = 0$.

The proposed local-global blending is aimed at estimating the optimal parameter K for vertex partitioning by means of the gradual blending between two opposite strategies: **i)** finest partitioning into 1-ring blocks obtained for K=0 [NSZ18]; **ii)** global optimization obtained, when K reaches the maximal deviation $\Delta_{\max}$ in the descent direction over the neighboring vertices. Starting with $K = 0$ and $K = \Delta_{\max}$ at the first iterations, we steadily update the partitioning threshold according to the observed progress for achieving an optimal number of coordinate blocks. In unconstrained problems, initialized by valid maps, our algorithm behaves exactly as an integrated core-solver (BQCN, AKVF, CM, etc.), except of a very short period, typically 2-3 first iterations for which computational cost is negligible. In the presence of highly-distorted invalid initializations, ABCD passes through two phases: the first one of cleaning foldovers, where distortions are optimized in coordinate blocks of varying sizes, and the stage of optimizing a positively oriented map, during which ABCD typically behaves as a global solver.

The proposed *displacement norm* is the magnitude of non-rigid motion by which target vertices are moved in successive iterations:

$$\text{Disp}^i = \left\| \boldsymbol{x}^i - \boldsymbol{x}^{i-1} - \text{Proj}_{\text{Ker}(\boldsymbol{x}^{i-1})} \left(\boldsymbol{x}^i - \boldsymbol{x}^{i-1} \right) \right\|_{\text{Fro}}, \tag{3}$$

where $\text{Ker}\left(\boldsymbol{x}^{i-1}\right)$ denotes the linear space of rigid transformations of the target shape at iteration $i-1$. Disp^i is employed as performance estimator for improving convergence criteria and for controlling the local-global blending.

3. Results

Providing a locally-injective initialization was until recently one of the major limitations in geometric optimization. We show that our algorithm overcomes these obstacles over a wide range of scenarios that include: 2D and 3D shape deformations, Tutte embedding onto nonconvex domains and constrained parametrization (see Fig. 1). According to our tests, ABCD exhibits superior robustness over existing map fixers, both with respect to the number of invalid simlices, distortion level and complexity of positional constraints. We intend to further explore Tutte mapping into nonconvex regions as an alternative starting point for accelerating parametrizations. Such methods can be initialized by parameterizing a low-resolution shape, and their performance can be boosted by the recent acceleration techniques and embedding methods.

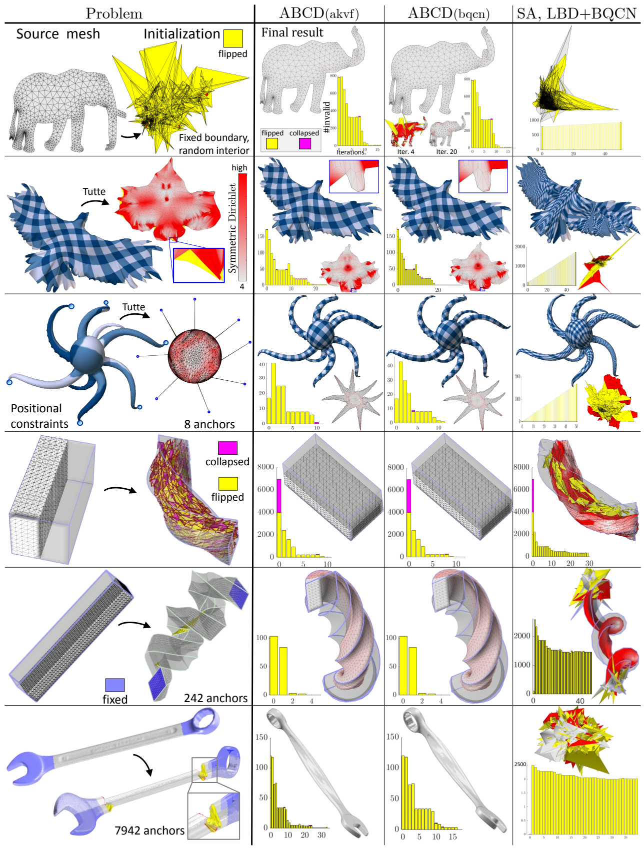

Figure 1: *Comparing ABCD, based on AKVF and BQCN core-solvers, with SA in 2D (rows 1-3) and LBD+BQCN in 3D (rows 4-6). The results include numbers of invalid simplices per iteration.*

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