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A Differentiable Material Point Method Framework for Shape Morphing

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Abstract

We present a novel, physically-based morphing technique for elastic shapes, leveraging the differentiable material point method (MPM) with space-time control through per-particle deformation gradients to accommodate complex topology changes. This approach, grounded in MPM's natural handling of dynamic topologies, is enhanced by a chained iterative optimization technique, allowing for the creation of both succinct and extended morphing sequences that maintain coherence over time. Demonstrated across various challenging scenarios, our method is able to produce detailed elastic deformation and topology transitions, all grounded within our physics-based simulation framework.

CCS Concepts
• ***Computing methodologies*** $\rightarrow$ ***Animation; Physical simulation;***

1. Introduction

Keyframe control and shape interpolation are essential techniques in animation, used to create smooth transitions for characters and objects. Traditionally, shape morphing algorithms focus on optimization, balancing constraints from keyframed shapes with additional terms to ensure physical realism. These terms may involve optimal transport or principles like Newton's second law. There are two main approaches: fluidic physical models, which handle topology changes well with external forces, and elasticity-based methods, which create visually appealing deformations with internal forces but usually assume a constant topology.

In this research, we combine the strengths of both approaches by introducing a physically-based shape morphing algorithm for elastic materials. This method supports large deformations and dynamic topology changes, using a differentiable material-point method (MPM) simulator optimized for elastic materials and controlled through deformation gradients. Inspired by the preliminary work found in [XL23], our approach includes a multi-pass optimization framework and a differentiable Eulerian loss function for unique topological transformations. The present work solves the instability issues identified in [XL23] and enhances overall performance. These results demonstrate significant deformations and dramatic topology changes.

[†] Equal contribution.

2. Method

2.1. Differentiable Simulation

In this study, we implemented MLS-MPM, which is naturally differentiable; analytical gradients have been derived by Hu et al. [HLS*19]. We manually implemented the necessary analytical gradients, to have more control over the implementation and avoid potential issues with software-based automatic differentiation [JMJF23]. In addition to gradients, a differentiable simulator naturally requires storing the feedforward simulation network to allow for the backpropagation of derivatives. We implemented this network by storing the computation graph of each timestep layer $\mathbf{T}^n$, which can be abbreviated by the key steps with superscript denoting the timestep number:

$$\mathbf{T}^n = \mathbf{P1}^n \rightarrow \mathbf{P2G}^n \rightarrow \mathbf{G}^n \rightarrow \mathbf{G2P}^n \rightarrow \mathbf{P2}^n. \tag{1}$$

Then, backpropagation can then be used on the simulation network to compute gradients via the chain rule:

$$\mathbf{T}^1 \leftarrow \cdots \leftarrow \mathbf{T}^n \leftarrow \mathcal{L}. \tag{2}$$

Our method involves controlling particles' deformation gradients, so we introduce a control deformation gradient term separate from the time-evolved deformation gradients.

2.2. Deformation Gradient Control

To represent morphing animation, which fundamentally requires handling large deformations and accurate rotations, we have chosen

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fixed co-rotational elasticity [SHST12, JST*16] as the constitutive model. In elastic constitutive models, the deformation gradient $\mathbf{F}$ is used to ensure that internal energy is only generated when $\mathbf{F}$ indicates a non-rigid transformation between the initial state $\mathbf{X}$ and the deformed state $\mathbf{x}$. Therefore, we define our elastic energy function with the Piola-Kirchoff stress tensor as follows:

$$\mathbf{P}(\mathbf{F}) = 2\mu(\mathbf{F} - \mathbf{R}) + \lambda(J-1)J\mathbf{F}^{-T}, \tag{3}$$

where J is the determinant of $\mathbf{F}$ and $\mathbf{R}$ is its rotational component, which can be computed via the polar decomposition of $\mathbf{F}$ [GFJT16].

Using this elastic model, we can manually control internal forces in our MPM simulation through the particle deformation gradients. Of course, to actually drive a shape morph, an automatic method for per-particle deformation gradient control is required. We achieve this through the optimization of the feedforward simulation network, which requires a loss function to be defined.

2.3. Log-based mass loss function

In MPM, we utilize a background grid whose nodes store interpolated quantities like mass from particles. Using a nodal mass loss function instead of a point position loss function simplifies input geometry requirements, such as the need for identical numbers of particles in the input and target geometries.

We compute the sum of the squared differences between the nodal masses m_i and m_i^* of the interpolated input and target masses:

$$\mathcal{L} = \sum_i \frac{1}{2}(m_i - m_i^*)^2, \tag{4}$$

However, this loss function can cause "mass ejections", especially with large shape differences, due to local minima. These mass ejections result from stronger forces in areas with less overlap between input and target geometries (Please refer to [XL23] for more details). To mitigate this, we propose a log-based mass loss function to reduce sensitivity to outliers.

$$\mathcal{L} = \sum_i \frac{1}{2}(\ln(m_i + 1) - \ln(m_i^* + 1))^2. \tag{5}$$

3. Results

We use a modified explicit time integration scheme for MLS-MPM, which helps prevent instability when taking larger timesteps. MPM point cloud visualization was performed using Polyscope [S*19], and rendering was conducted using Houdini. We ran the experiment on a machine with an Intel i7-10750H CPU, 32GB RAM, and an NVIDIA RTX 2060 GPU

4. Conclusions and Future Work

This study presented a framework for shape morphing using the differential material point method (MPM), emphasizing the novel log-based nodal mass loss function to prevent spurious particle movement and enable efficient, detailed morphing animations. Future work will focus on improving the parallel performance of our method, exploring GPU-based implementations, and enhancing scaling up to a few dozen cores for practical animation use cases.

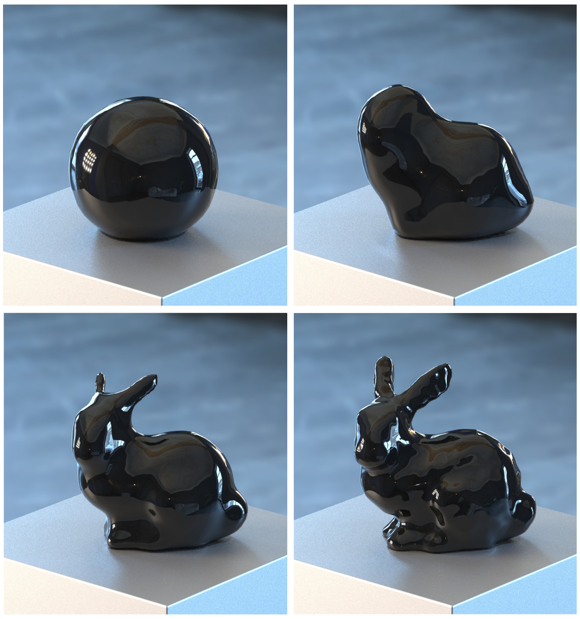

Figure 1: *Sphere to bunny morphing animation. Key stages show the sphere evolving into a detailed bunny form, highlighting our method's precision in capturing sharp features like the ears.*

References

[GFJT16] Gast T. F., Fu C., Jiang C., Teran J.: Implicit-shifted Symmetric QR Singular Value Decomposition of 3x3 Matrices. URL: https://api.semanticscholar.org/CorpusID:11952786. 2

[HLS*19] Hu Y., Liu J., Spielberg A., Tenenbaum J. B., Freeman W. T., Wu J., Rus D., Matusik W.: Chainqueen: A Real-Time Differentiable Physical Simulator for Soft Robotics. In *2019 International Conference on Robotics and Automation (ICRA)* (2019), pp. 6265–6271. URL: https://doi.org/10.1109/ICRA.2019.8794333, doi:10.1109/ICRA.2019.8794333. 1

[JMJF23] Johnson D., Maxfield T., Jin Y., Fedkiw R.: Software-Based Automatic Differentiation is Flawed. *arXiv preprint arXiv:2305.03863* (2023). URL: https://doi.org/10.48550/arXiv.2305.03863, doi:arXiv:2305.03863v1. 1

[JST*16] Jiang C., Schroeder C., Teran J., Stomakhin A., Selle A.: The Material Point Method for Simulating Continuum Materials. In *ACM SIGGRAPH 2016 Courses* (New York, NY, USA, 2016), SIGGRAPH '16, Association for Computing Machinery. URL: https://doi.org/10.1145/2897826.2927348, doi:10.1145/2897826.2927348. 2

[S*19] Sharp N., et al.: Polyscope, 2019. URL: www.polyscope.run. 2

[SHST12] Stomakhin A., Howes R., Schroeder C., Teran J. M.: Energetically Consistent Invertible Elasticity. In *Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation* (Goslar, DEU, 2012), SCA '12, Eurographics Association, p. 25–32. URL: https://dl.acm.org/doi/10.5555/2422356.2422361, doi:10.5555/2422356.2422361. 2

[XL23] Xu M., Levin D. I.: Deformation Gradient Control of Physically Simulated Amorphous Solids. In *Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation* (New York, NY, USA, 2023), SCA '23, Association for Computing Machinery. URL: https://doi.org/10.1145/3606037.3606840, doi:10.1145/3606037.3606840. 1, 2