Appearance of Interfaced Lambertian Microfacets
using STD Distribution

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BRDF Models

Many existing models [Phong, Ward, CT82, ON94, Ash00, Jak14, Wu15, Bel17, etc.]
- Only few parameters, more or less intuitive and easy to control
- Some are designed specifically for fitting parameters

Some of them aim designed for physically-based applications
- (Energy conservation and reciprocity)
  - ⇒ Microfacet-based models often employed

$$L(i, o, n) = \frac{d^2 \phi_o(i, o)}{dS \cos \theta_o d\omega_o}$$

$$f(i, o, n) = \frac{dL(o, n)}{dE(i, n)}$$
Microfacet Representation

General Equation [ON94,Walt07]:

\[
f(i, o, n) = \int_{\Omega_+} \left| \frac{\mathbf{im}}{\mathbf{in}} \right| f^\mu(i, o, m) \left| \frac{\mathbf{om}}{\mathbf{on}} \right| D(m) G(i, o, m) d\omega_m. \tag{1}
\]

⇒ All microfacets may contribute
⇒ Rough surfaces imply multiple light reflections

Simplifies with specular microfacets \( f^\mu \) [TS67,CT82,Walt07]:

\[
f(i, o, n) = \frac{F(i, h) D(h) G(i, o, h)}{4|\mathbf{in}||\mathbf{on}|}, \tag{2}
\]

⇒ Only one microfacet orientation can contribute
⇒ Multiple light reflections are ignored

Many authors have discussed:

- Relationships between D and GAF [TS67,Ash00,SB,Heitz,etc.]
- Energy conservation with specular microfacets [Kel01,TVCG17]
- Multiple scattering [Heitz,TVCG17]
Microfacet Representation

- Playing with $f^\mu$ offers a large panel of different materials.
Microfacet Representation

- Playing with $f^\mu$ offers a large panel of different materials.
- Geometrical Attenuation Factor (GAF).

Torrance-Sparrow (V-cavity profile)
Smith-Bourlier (Uncorrelated microfacets)
Microfacet Representation

- Playing with $f^\mu$ offers a large panel of different materials.
- Geometrical Attenuation Factor (GAF).
- Normal Distribution Functions.

Beckmann distribution
GGX or Trowbridge-Reitz
Microfacet Representation

- Playing with $f^\mu$ offers a large panel of different materials.
- Geometrical Attenuation Factor (GAF).
- Normal Distribution Functions.
- Multiple scattering between microfacets.

(image from [Heitz16])
Interfaced Lambertian (IL) Model [TVCG17]

Several observations can be made:

- The glossy term increases according to incidence angle
- Thus, a constant Lambertian term is not adapted to energy conservation
- Solution: Rough Lambertian background covered with a flat Fresnel interface

\[
\frac{1}{\pi n_i^2} T(i, m) T(o, m) \frac{K_d}{(1 - K_d r_i)} ,
\]

\( r_i \) for multiple scattering (analytical cf. [TVCG17])
Flat IL Material

- Flat surface: Analytical representation, including multiple light scattering
- Body term decreases according to incidence angles, and specularity
- Decreases also at grazing observation angles

\[
\begin{array}{c|c|c|c}
\theta_i & n_i = 1.0 & n_i = 1.2 & n_i = 1.33 & n_i = 1.5 \\
\hline
0^\circ & \\ \\
45^\circ & \\
90^\circ & \\
\end{array}
\]
Rough IL Material

The general BRDF equation should be integrated, with:

\[
f(i, o, n) = \int_{\Omega^+} \frac{|im|}{|in|} \left[ f_s^\mu(i, o, m) + f_b^\mu(i, o, m) \right] \frac{|om|}{|on|} D(m) G(i, o, m) d\omega_m
\]  

(3)

- The first integral corresponding to \( f_s \) corresponds to the glossy term

\[
f_s(i, o, n) = \frac{F(i, m)D(m)G(i, o, m)}{4|in||on|},
\]

- The second term \( f_b \) has no analytical solution

Monte Carlo for the rendering Equation:

\[
L_o(x, o, n) = L_e(x, o, n) + \int_{\Omega^+} L_i(x, i, n) f(i, o, n)|in|d\omega_i,
\]  

(4)

where \( f \) is given by Equation 3, which includes

\[
f_b^\mu(i, o, n) = \frac{1}{\pi n_i^2} T(i, m) T(o, m) \frac{K_d}{(1 - K_d r_i)}
\]  

(5)
Rough IL Material

Solution: use Monte Carlo process again.
- Importance sampling of one microfacet for the body term
- Slightly increases noise (since increases integral dimension)
- But allows to handle multiple scattering between microfacets [Heitz16,TVCG17]

⇒ Inherently accounts for anisotropy, given anisotropic distributions
Appearance

General model, accounts for:

- Flat Lambertian \( (\sigma = 0.0, n_i = 1.0) \)
- Rough Lambertian \( (n_i = 1.0) \), with backscattering
- Rough dielectric mirrors \( (K_d = 0.0) \)
- Rough interfaced Lambertian (general case)

⇒ Illustrated on next slide

An approximate model is proposed in [TVCG17], with:

- Beckmann and Gauss distributions
- Torrance-Sparrow’s GAF

⇒ Makes it possible to use with interactive applications and fitting

Note that:

- Surface and substrate roughnesses are the same
- Light scattering between microfacets should be handled
IL BRDF lobes

Distributions and GAFs for various values of $n_i$ and $\sigma$, illustrated at $\theta_i = 60^\circ$ (log scale).
With Beckmann Distribution and Smith GAF

\( n_i = 1.5 \)

\( n_i = 1.33 \)

\( n_i = 1.2 \)

\( n_i = 1.0 \)

\( \sigma = 0.001 \)

\( \sigma = 0.005 \)

\( \sigma = 0.1 \)

\( \sigma = 0.3 \)
IL BRDF lobes: approximate model

Comparison between Monte Carlo BRDF estimation of Lambertian (L) and interfaced Lambertian (IL) materials and our approximate model, with Gaussian (G) and Beckmann (B) distributions, and Torrance-Sparrow (TS) GAF (log scale).
Discussion

- Management of metals (conductors) ?
  ⇒ *Nothing new [CT82], since almost no transmission*

- Generalization of approximate models ?
  ⇒ *much more complicated...*
  ⇒ *Approximation relies on both D and G*
  ⇒ *Our method extends [ON94], based on Gaussian/Beckman distributions*

- Generalization of distribution and GAF
  - Many existing distributions
  - Without analytical GAF and/or analytical importance sampling
    ⇒ *This presentation provides some results with STD (next slides)*

- Management of light scattering between microfacets
  - Two existing contributions: [Heitz16] with SB GAF; [TVCG17] with TS GAF
  - Path tracing implementation
    ⇒ *Both applied to STD and IL in this presentation*
### Student’s T-Distribution

Introduced at EG 2017 [EG17]:

\[
D^{STD}(m) = \frac{(\gamma - 1)^{\gamma} \sigma^2 \gamma - 2}{\pi \cos^4 \theta_m (\gamma - 1) \sigma^2 + \tan^2 \theta_m)^\gamma}
\]  \hspace{1cm} (6)

- Inspired from GTR (Generalized Towbridge Reitz) [TR75, Walter07]
- Includes both GGX and Beckmann’s distributions
- With analytical GAF formulation following the Smith’s formulation
- With analytical importance sampling

![Graphs showing the distribution for different values of \(\sigma\) and \(\gamma\)]
Influence on appearance

⇒ Anisotropy also handled (rough aluminium in this case)
Influence on appearance

Visual impact of STD

\[ \gamma = 2 \quad \sigma = 0.01 \]
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 2 \quad \sigma = 0.1 \]

- \( \gamma = 2 \) (GGX)
- Beckmann
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 2 \quad \sigma = 0.2 \]

- \( \gamma = 2 \) (GGX)
- Beckmann
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 2 \quad \sigma = 0.3 \]

- \( \gamma = 2 \) (GGX)
- Beckmann
Influence on appearance

Visual impact of STD

\[ \gamma = 2 \quad \sigma = 0.4 \]
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 2 \quad \sigma = 0.5 \]
Influence on appearance

Visual impact of STD

\[ \gamma = 2 \quad \sigma = 0.6 \]
Influence on appearance

Visual impact of STD

$\gamma = 2 \quad \sigma = 0.7$
Influence on appearance

Visual impact of STD

\[ \gamma = 2 \quad \sigma = 0.9 \]
Influence on appearance

Visual impact of STD

\[ \sigma = 0.9 \]
Influence on appearance

Visual impact of STD

Surface profile

\[ \sigma = 0.7 \]

- \( \gamma = 2 \) (GGX)
- Beckmann
Influence on appearance

Visual impact of STD

Surface profile

\[ \sigma = 0.6 \]

\( \gamma = 2 \) (GGX)

Beckmann
Influence on appearance

Visual impact of STD

Surface profile

$\sigma = 0.5$

- $\gamma = 2$ (GGX)
- Beckmann
Influence on appearance

Visual impact of STD

Surface profile

\[ \sigma = 0.4 \]

- \( \gamma = 2 \) (GGX)
- Beckmann
Influence on appearance

Visual impact of STD

Surface profile

\[ \sigma = 0.3 \]

\( \gamma = 2 \) (GGX)

Beckmann
Influence on appearance
Influence on appearance

Visual impact of STD

Surface profile

\[ \sigma = 0.1 \]

\[ \gamma = 2 \text{ (GGX)} \]

Beckmann
Influence on appearance

Visual impact of STD

Surface profile

\[ \sigma = 0.01 \]

- \( \gamma = 2 \) (GGX)
- Beckmann
Influence on appearance

Visual impact of STD

Surface profile

$\sigma = 0.3$

$\gamma = 2$ (GGX)

Beckmann
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 20 \quad \sigma = 0.3 \]
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 10 \quad \sigma = 0.3 \]
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 6 \quad \sigma = 0.3 \]

- \( \gamma = 2 \) (GGX)
- \( \gamma = 6 \)
- Beckmann
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 4 \quad \sigma = 0.3 \]
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 3 \quad \sigma = 0.3 \]
Influence on appearance

Visual impact of STD

Surface profile

$\gamma = 2$, $\sigma = 0.3$
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 1.75 \quad \sigma = 0.3 \]
Influence on appearance

Visual impact of STD

Surface profile

\[ \gamma = 1.51 \quad \sigma = 0.3 \]
Discussion

Advantages of STD:

- Accurate control of roughness
- Interesting use for fitting (combines the advantages of GGX and Beckmann)
Fitting with STD

log(\text{fit. error})

RMSE (rendering)
Discussion

Advantages of STD:

- Accurate control of roughness
- Interesting use for fitting (combines the advantages of GGX and Beckmann)
- Provides a general tool for choosing distribution
Discussion

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Advantages of combining IL with STD:
- Accounts for a physical representation of body scattering
- Combines advantages of both
- Further generalizes both
Discussion

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- Accounts for a physical representation of body scattering
- Combines advantages of both
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Implementation issues:
- Does not make any difference for IL
- Possible to include the combination in any Monte Carlo rendering system
- Also possible to handle multiple scattering
Influence on appearance

According to $\gamma$, with two different roughnesses $\sigma$ (Smith GAF with $n_i = 1.5$):

- $\sigma = 0.1$, $\gamma = 1.55$
- $\sigma = 0.1$, $\gamma = 8$
- $\sigma = 0.3$, $\gamma = 1.55$
- $\sigma = 0.3$, $\gamma = 8$
Influence on appearance

When changing GAF ($\gamma = 1.75$, $n_i = 1.5$ and $\sigma = 0.7$):

For grazing observation angles:

- Torrance-Sparrow’s GAF tends to overestimate gloss [Heitz14]
- Glossy effects remain high despite increasing roughness
Influence on appearance

Comparisons with and without multiple scattering between microfacets:

- Rough Lambertian \((n_i = 1.0)\)
- \(\gamma = 8, \sigma = 0.7\)
- Smith-Bourlier GAF
Influence on appearance

Comparisons with and without multiple scattering between microfacets:

- Interfacet Lambertian microfacets ($n_i = 1.5$)
- $\gamma = 1.75, \sigma = 0.5$
- Smith-Bourlier GAF
Conclusion and Future Work

STD with interfaced Lambertian microfacets:

- Physically based model
- Management of specular and body reflections
- Only few parameters
- Extends the range of rendered materials

Future work:

- Better STD importance sampling
  ⇒ *What about Visible Normals Importance Sampling?*

- In depth fitting analysis
- Correlation between the interface and the substrate roughness in IL
- Any other idea?