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Statistical Characterization of Surface Reflectance

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Abstract

The classification of surface reflectance functions as diffuse, specular, and glossy has been introduced by Heckbert more than two decades ago. Many rendering algorithms are dependent on such a classification, as different kinds of light transport will be handled by specialized methods, for example caustics require specular bounce or refraction. Due to the increasing wealth of surface reflectance models including those based on measured data, it has not been possible to keep such a characterization simple. Each surface reflectance model is mostly handled separately, or alternatively, the rendering algorithm restricts itself to the use of some subset of reflectance models. We suggest a characterization for arbitrary surface reflectance representation by standard statistical tools, namely normalized variance known as Squared-Coefficient-of-Variation (SCV). We show by videos that there is even a weak perceptual correspondence with the proposed reflectance characterization, when we use monochromatic surface reflectance and the images are normalized so they have the unit albedo.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture I.4.1 [Image Processing and Computer Vision]: Digitization and Image Capture—Reflectance

1. Introduction

The surface reflectance has been studied in many scientific fields including computer graphics, computer vision, optics, remote sensing, visual appearance etc. The formalization of surface reflectance known as *bidirectional reflectance distribution function* (BRDF) [NRH*77] is in the core of the *rendering equation* [Kaj86]. It is known that some classes of rendering algorithms are suited for some surface reflectance; i.e. the recursive ray tracing is efficient for mirror-like surfaces, while radiosity is efficient for diffuse surfaces. These limitations of rendering algorithms made the research oriented to general solutions that ideally would work independently on the surface reflectance used. In many research papers and applications the algorithms pose these limitations in form of exponent of Phong model or specular roughness of Ward model. Sometimes the limitations are specified in papers vaguely in terms as ‘almost diffuse, moderately glossy’, highly glossy etc. However, such specification of the surface reflectance characterization is crucial for both visual quality of images and performance of many rendering algorithms.

In our paper we propose a simple but powerful characterization of surface reflectance that can be used in rendering algorithms irrespective if they are biased, consistent, or unbiased. We use for our analysis the rendering equation and suggest that a meaningful characterization for classification of surface reflectance is its normalized variance defined carefully for sampling $1/\pi \cos\theta$ distribution. We advocate to use standard statistical tools instead of ad hoc approaches when such a characterization is needed in rendering algorithms.

2. Motivation and Related Work

Our motivation came upon implementation of some algorithms such as photon mapping [Jen01] that distinguishes between more types of light transport to deal with them efficiently based on the surface reflection. The question is if to store photons on the surfaces that shall be ‘almost diffuse’. A similar problem is if to store virtual point light sources in the context of many-lights methods [DKH*13]. The algorithm needs a classification to distinguish which surfaces are ‘diffuse or almost diffuse’ to allow to store photons on these surfaces. Heckbert [Hec90] provides the classification to the surface reflectance on diffuse, mirror and everything else as glossy, with possible separation of BRDF to its diffuse and glossy parts. He also provides the description of glossy parts as that is inside the cone in direction of ideal reflection. In practice the problem is not that sim-

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ple, when we deal with real-world measured data. There are some options how to solve the problem. First, we can use methods such as independent component analysis to factor the BRDF into two parts and use diffuse part and the rest directly. This is costly and it also does not give any answer how much diffuse is the surface reflectance. There are attempts for the answer, such as the measure of diffuseness $r_d = \rho_{diffuse}/(\rho_{diffuse} + \rho_{specular})$ by Balling and Marciniak [BM09]. Second, we can fit the data to a chosen analytical BRDF model and use for our discrimination the parameters of the model such as often used exponent or specular roughness for a particular BRDF model.

In general, fitting the data to a single BRDF model and using its selected parameter as some characterization is not possible for several reasons. First, some measured data including anisotropic ones or BTF, SVBRDF, or BTF data can have several modalities, that can come also from spatial filtering. The multi-modality can be captured by some multi-lobe BRDF models such as Lafortune [LFTG97] or in the context of bidirectional texture functions by a mixture of functions as in the model by Wu et al. [WDR11]. Second, it is unclear how to put together the specular roughness from individual lobes. Third, even for a unimodal shaped distribution such as physically corrected Phong model [LW94] it is not unclear how to deal with it, since for a single lobe there is its shape given by exponent and specular albedo and diffuse albedo. Some analytical BRDF models can be fitted better to some measured data than the others [NDM05]. Some analytical models do not have explicit diffuse parts, for some BRDF models fitting is prohibitively expensive to be used in rendering algorithms. In practice (such as [CPF10]) it is common to restrict the number of BRDF models used to a few and then use some thresholds in algorithms that work acceptably for selected BRDF models. The design of such ad hoc approaches is well motivated but not theoretically backed up or driven by well justified objectives.

Appearance and perception. The problem how much glossy the material looks like has been dealt with from the viewpoint of perception since 1930, more detailed survey is available in [Van09]. There is a physical gloss measured by special gantries called glossmeters and there are tens of definitions how the gloss should be measured in dependence on the industry and application scenario. Further, it was documented that there are several different gloss definitions useful for different industries already in year 1937: specular gloss, contrast gloss, sheen, absence-of-bloom gloss or haze, distinctness-of-image (DOI) gloss, surface-uniformity gloss. In computer graphics Pellacini et al. [PFG00] introduces a new BRDF model that is motivated by gloss reception. Matusik et al. [MPBM03] provides analysis of gloss space for 100 measured isotropic materials. More recent advances and citations to older work focused on gloss are surveyed in the theses of Vangorp [Van09] and LeLoup [Lel12].

Image Statistics, BRDF measurements, Optics. Dror et al. [DAW01] use image statistics to estimate BRDF from images under natural illumination and how to apply it to estimate specular roughness of Ward BRDF model. This is then improved by Ghosh et al. [GCP*09] by the use of orthogonal spherical basis, where specular roughness of Ward model corresponds to normalization and scaling of standard deviation. There is also a concept of surface roughness used in optics (for example by Hoover and Gamiz [HG06]) as $\sigma = \sigma_h/\lambda$, where σ_h is the standard deviation of the surface height distribution and λ is the illumination wavelength.

Importance Sampling in Rendering Algorithms. In addition to the above mentioned notation by Heckbert [Hec90] Veach and Guibas [VG95] and Veach in his thesis mention the concept of specular roughness when developing multiple importance sampling (MIS), we quote [Vea97, page 253]:

> " In particular, we consider spherical light sources of varying radii, and glossy materials that have a *surface roughness parameter (r)* that determines how sharp or fuzzy the reflections are. Smooth surfaces ($r = 0$) correspond to highly polished, mirror-like reflections, while rough surfaces ($r = 1$) correspond to diffuse reflection. It is possible to simulate a variety of surface finishes by using intermediate roughness values in the range $0 < r < 1$."

Then the roughness is used in Phong-model as exponent $\alpha = 1/r - 1$, and this is used to report the results for testing MIS. Pajot et al. [PBPP11] presents a concept of 'representativity' to improve on the robustness of estimators when using MIS in numerical integration. They observed the cases when the sampling leads to high variance and propose an empirical measure to deal with the problem. From the two representativity measures presented in the paper the empirical measure based on BRDF is called 'directionality' and consists of three parts, diffuse d_d, specular d_s, and mirror-like d_t. They give the derivation of directionality for physical Phong model. For diffuse part $d_d = 1/(2.\pi)$, for glossy part of roughness of physical Phong model as $d_s = 1/2\pi + (1 - 1/2\pi)(\pi/3 - \cos^{-1}((1/2)^{\frac{1}{n+\alpha}})).3/\pi$. For mirror-like interaction $d_t = 1$. The three terms are composed by albedo for the three parts to a single term $R(f_r(x, \omega_i, \omega_o))$ that is zero for diffuse BRDF and one for mirror-like surface. This proposal on representativity is not theoretically backed up and cannot be generalized to arbitrary surface reflectance.

In addition to photon mapping [Jen01], the need for concept to quantify either diffuseness or glossiness to drive the computation is used for example by Dammertz et al. [DKL10]. They deal with the similar concept as Veach, given characterization $r = 0$ for diffuse and $r = 1$ for specular for exponent of physically based model. They use however proportional formulation in the form of physical Phong model the exponent $\alpha = 1024.r$ and threshold $r_T = 0.2$, but this is also ad hoc formula not theoretically founded.

The practitioners deal with the problem of surface reflectance characterization by using empirical functions derived from running the computation and using the tested values or ad hoc formulas for different algorithms. Even for a simple case such as environment map illumination it is needed and shown to be efficient by Colbert et al. [CPF10]. Their proposed solution is valid only for a single preselected BRDF model restricting parameters to a one or two.

2.1. BRDF properties

BRDF (i.e. bidirectional reflectance distribution function) is defined by Nicodemus [NRH*77] by means of the reflectance equation:

$$f_r(x,\omega_o,\omega_i) = \frac{dL(x \to \omega_o)}{L(x \leftarrow \omega_i).\cos(\theta_i)d\omega_i} = \frac{dL(x \to \omega_o)}{L(x \leftarrow \omega_i).(\omega_i.\vec{n})d\omega_i} \quad (1)$$

In this definition it fulfills Helmholtz reciprocity given as $f_r(x,\omega_o,\omega_i) = f_r(x,\omega_i,\omega_o)$. Observe that $f_r(x,\omega_o,\omega_i)$ is not in principle a probability density function (as it does not integrate to one with incoming directions ω_i, but see below), it is unit-less (i.e. unit is [sr^{-1}], but this is still unit-less as steradian is unit-less). To obtain a probability density function (pdf) from the BRDF, it should be redefined using the reflectance equation. We can define it in the following way, for fixed ω_o and integrating over incoming directions ω_i.

$$\text{pdf}(x,\omega_o,\omega_i) = \frac{1}{a(x,\omega_o)}.f_r(x,\omega_i,\omega_o).(\omega_i.\vec{n}), \quad (2)$$

where albedo $a(f_r(x,\omega_o))$ for fixed outgoing direction ω_o is defined as:

$$a(f_r(x,\omega_o)) = \int_\Omega f_r(x,\omega_i,\omega_o).(\omega_i.\vec{n})d\omega_i \quad (3)$$

And then:

$$\int_\Omega \text{pdf}(x,\omega_o,\omega_i)d\omega_i = \frac{1}{a(x,\omega_o)} \int_\Omega f_r(x,\omega_i,\omega_o).(\omega_i.\vec{n})d\omega_i = 1 \quad (4)$$

Observe that albedo $a(f_r(x,\omega_o))$ can be considered as π times the expected value of $f_r(x,\omega_i,\omega_o)$ when sampling with pdf $(\omega_i.\vec{n}).\frac{1}{\pi}$

$$a(f_r(x,\omega_o)) = \pi \int_\Omega f_r(x,\omega_i,\omega_o).\frac{1}{\pi}(\omega_i.\vec{n})d\omega_i = \pi E[f_r(x,\omega_i,\omega_o)] \quad (5)$$

3. Proposal of BRDF characterization

We can analyze the impact of BRDF to the variance of a result when computing rendering equation under simplified conditions, in our case using the reflectance equation in eq. 1 (i.e. local illumination by environment map $I(\omega_i)$) as:

$$L(x,\omega_o) = \int_\Omega I(\omega_i)f_r(x,\omega_i,\omega_o)(\omega_i.\vec{n})d\omega_i \quad (6)$$

In rendering algorithms we aim to get the lowest variance of the estimator derived from the rendering equation such as above. This is often achieved by MIS [VG95], drawing samples according to two or more functions in the product. In eq. 6 we have a product of three functions, $I(\omega_i)$, $f_r(x,\omega_i,\omega_o)$, and $(\omega_i.\vec{n})$. There is an interesting case when the product can be seen as $f_1(x).f_2(x)$ for $f_1(x) = f_r(x,\omega_i,\omega_o)$ and $f_2(x) = I(\omega_i).(\omega_i.\vec{n}) = \omega_i.\vec{n}$. The variance $V[L(x,\omega_o)]$ when sampling from $f_2(x)$ in eq. 6, for a slowly varying environment map $I(\omega_i)$, approaches the variance of BRDF function, being equal in the limit when $I(\omega)$ is constant, i.e., $V[L(x,\omega_o)] = V[f_r(x,\omega_i,\omega_o)]$ for sampling according to $1/\pi(\omega_i.\vec{n})$. We can see that variance of $V[f_r(x,\omega_o)]$ plays a key role in value of V for this simplified analysis. Therefore we suggest to use the variance of BRDF in addition to albedo $a(f_r(x,\omega_o))$ that as we have seen corresponds to π times mean value $E[f_r(x,\omega_o)]$. This is more natural than measuring the ad hoc definition of diffuseness as used by some papers in the past.

To make the proposed variance characterization approximately comparable in perceptual domain for a pair or set of BRDFs, we suggest to use its normalized variant, known in statistics as in *Squared-Coefficient-of-Variation* (e.g. in [RK08], abbreviated as SCV). This then allows to visually compare perceived gloss if a pair of BRDFs are normalized by albedo. For short, we suggest to use the term *variant index of glossiness* (abbreviated as VIG) defined as:

$$r_g(f_r(x,\omega_o)) = \frac{V[f_r(x,\omega_o)]}{E^2[f_r(x,\omega_o)]} = \frac{E[(f_r(x,\omega_o) - E[f_r(x,\omega_o)])^2]}{E^2[f_r(x,\omega_o)]} \quad (7)$$
$$= \frac{\int (f_r(x,\omega_o) - \int f_r(x,\omega_o).(\omega_i.\vec{n})d\omega)^2.(\omega_i.\vec{n})d\omega_i}{(\int f_r(x,\omega_o).(\omega_i.\vec{n})d\omega)^2},$$

where variance $V[f_r(x,\omega_o)]$ and mean $E[f_r(x,\omega_o)]$ are computed according to $1/\pi(\omega_i.\vec{n})$ distribution. Its properties are then so that for diffuse BRDF $r_g = 0$, for mirror $r_g = \infty$. Since we know also albedo $a(f_r(x,\omega_o)) = \pi E[f_r(x,\omega_o)]$, the non-normalized variance $V[f_r(x,\omega_o)]$ can be easily computed when needed from $r_g(f_r(x,\omega_o))$.

We expect that the statistics above can be computed in closed form for analytical BRDF models as we show in appendix on an example. For measured tabulated data such as Matusik [MPBM03] it is possible to compute both $r_g(x,\omega_o)$ and $a(f_r(x,\omega_o))$ numerically in precomputation and store them to a 1D (isotropic BRDF) or 2D (anisotropic BRDF) table similar to BRDF data, as this table has two dimensions less than the data the overhead storage is negligible. The values for different ω_o can be then interpolated from the values in table. Note that the same characterization is computable for other used surface reflectance models, where Helmholtz reciprocity does not hold, such as BTFs. After normalization $r_g(f_r(x,\omega_o))$ must be computed numerically. We show $r_g(f_r(x,\omega_o))$ in four videos (available at web page `http://dcgi.felk.cvut.cz/~havran/REFLVIG/`) for rendering images of two objects (sphere and Stanford Bunny) using MERL BRDF data [MPBM03] illuminated by two environment maps (Grace Cathedral and St'Peters Basilica).

Limitations. For layered BRDF functions such as [WW07], where one BRDF is perfectly specular (such as lacquered wood), the proposed $r_g(f_r(x,\omega_o))$ has to be computed separately for layers since for variance mixing infinity to anything results in infinity. This corresponds to handling such BRDFs in rendering algorithms in practice. The perfect mirror reflections are handled by deterministically shooting a single ray in reflected directions, while diffuse and glossy layers are solved stochastically.

4. Conclusion

We have proposed a novel and easily computable surface reflectance characterization that can be utilized instead of ad hoc methods used to classify on reflection bounces in the past. Instead of using the characterization as a measure of 'diffuseness' in range from zero to one we show that for rendering algorithms is much more meaningful the measure of 'glossiness' computed as the variance of BRDF in range from zero (diffuse) to infinity (mirror) normalized by albedo for the sake of rough perceptual comparisons. We would like to advocate the use of standard statistical tools for surface reflectance models in applications and in future research.

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Appendix. We show here the formula $r_g(f_r(x,\omega_o))$ as defined by eq. 7 for physically based variant of Phong model presented by Lafortune [LW94], assuming the whole lobe is above the surface. The simplest case is when the outgoing direction ω_o is along the surface normal. The BRDF is:

$$f_r(\omega_o,\omega_i) = \rho_d/\pi + \rho_s \frac{n+2}{2\pi}.cos^n(\theta), \quad d\omega_i = \frac{sin\theta d\theta d\phi}{\pi} \quad (8)$$

For albedo $a(f_r(x,\omega_o)) = \pi.E[f_r(x,\omega_o)] = \rho_d + \rho_s$ then $r_g(f_r(x,\omega_o))$ can be then derived as:

$$r_g(f_r(x,\omega_o)) = \frac{V[f_r(x,\omega_o)]}{E^2[f_r(x,\omega_o)]} = \frac{\rho_s^2(\frac{(n+2)^2}{(2n+2)2} - 1)}{(\rho_d+\rho_s)^2} \quad (9)$$

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