A Visual Approach to Fair or Negotiated Resource Division

R. L. Ribler and J. Wise

University of Lynchburg, Department of Computer Science, Lynchburg, Virginia, United States

Abstract

The fair division problem addresses the frequently encountered situation in which a set of resources must be fairly divided between two or more stakeholders. Dividing possessions after a divorce, assigning tasks to workers, and determining the terms of contracts or treaties are all examples of this problem. Algorithms have been developed to provide solutions that optimize for various metrics, but for many reasons, including the lack of agreement on what constitutes fairness, algorithms cannot provide a definitive result. Visualizations, rather than providing a single candidate solution, can be used effectively to browse the search space and generate a pool of candidate allocations that are most likely to be appealing to all parties. Candidate solutions can be used by stakeholders, either separately or cooperatively, as the basis for negotiation. We demonstrate prototype software that provides this capability for a set of indivisible resources that are divided between two stakeholders.

CCS Concepts

• ***Human-centered computing*** → ***Information visualization; Visual analytics;***

1. Introduction

The *fair division* problem, in which a set of resources must be divided between two or more stakeholders, is a difficult problem that occurs frequently and at every level of importance. Common examples of the problem include dividing a couple's assets after a divorce, providing each heir a fair portion of an estate, and determining the schedules for a group of shift workers. More complex examples include contract negotiations and the establishment of treaties and agreements between nations.

There has been a lot of interesting work on the development of algorithms to address this problem. [BT96] [Mou19] [CCG*22] Most of the algorithms that have been developed ask each stakeholder to assign a value to every resource in the set.

These subjective values, sometimes referred to as *bids* are likely to be different for each stakeholder. The literature typically refers to stakeholders as *players*. A player's bid is converted to a percentage by dividing it by the sum of all of that player's bids.

After allocations have been determined for each player, a player can compute their subjective share of the resources as the sum of their successful bids. From that player's perspective, this is the percentage of the value of the resources that they have received. By exploiting the differences in subjective values across players, it is often possible to determine allocation in which each participant views the value of their allocation as more than their fair share, $1/p$, where p is the number of players.

For example, the *adjusted winner algorithm* described in [BT96] provides an example in which the algorithm is applied to the negotiation of the Panama Canal Treaty signed between the United States and Panama in 1977. Estimates of how much value each country placed on the individual resources are provided in the following table [Rai82]:

Issue	United States	Panama
US Defense Rights	22	9
Use Rights	22	15
Land and water	15	15
Expansion Rights	14	3
Duration	11	15
Expansion Routes	6	5
Compensation	4	11
Jurisdiction	2	7
US Military Rights	2	7
Defense of Panama	2	13

Applying the adjusted winner algorithm results in a division in which each stakeholder perceives that they have received approximately 66% of the value of the set. We can reproduce this result, and generate alternative candidate solutions quickly using visualization.

2. The Problems with Algorithms

While these algorithms are interesting intellectually, there are many practical problems that limit their application. There is often interdependency between the values of resources, so an item might be valued differently depending on the other resources in the allocation. Just as a monetary value cannot express all aspects of how something is valued, a single valuation is not necessarily sufficient to convey how a player values a resource. Finally, most algorithms

Proceedings published by Eurographics - The European Association for Computer Graphics.

DOI: 10.2312/evp.20241078

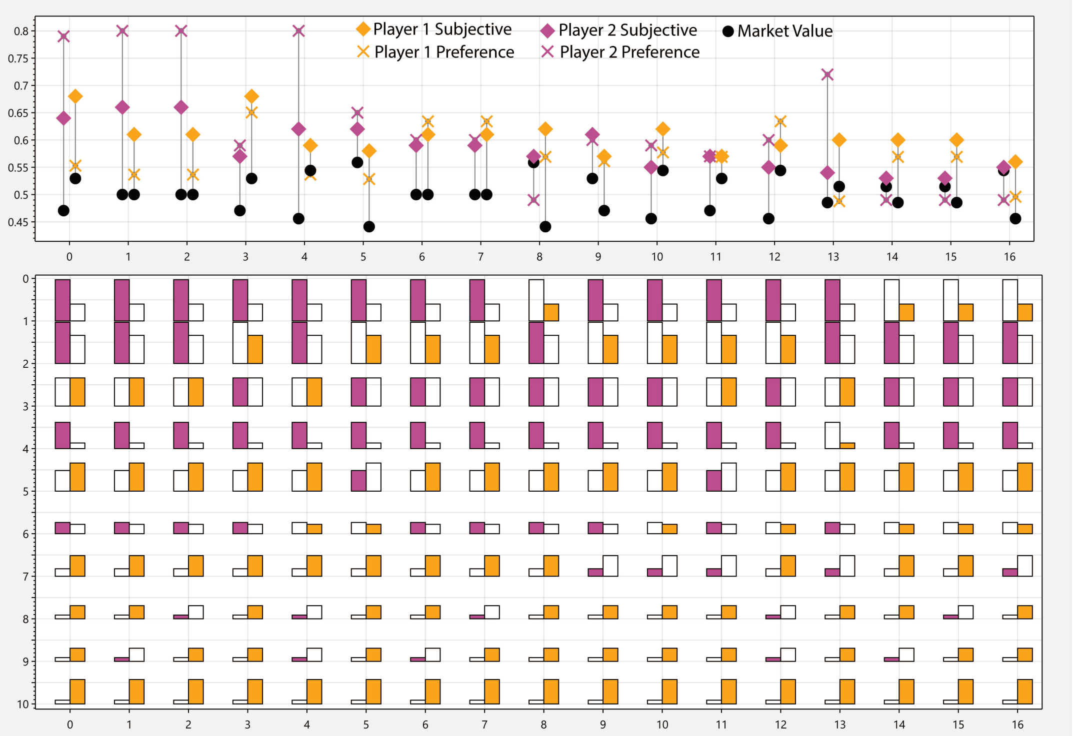

Figure 1: *Visualization of candidate allocations. Each pair of vertical lines in the upper panel represents a candidate allocation. Subjective values are plotted with a diamond, market values are plotted with a circle, and preference values are plotted with an X. The left member of the pair of lines provides the values for the first player, and the right member provides the values for the second player. The lower panel shows the allocations for each of the resources in the allocation in the corresponding upper panel display. Each pair of rectangles represents a resource. The rectangles on the left are shaded if the first player is allocated a resource, and the rectangles on the right correspond to the second players allocations. The height of the rectangle is proportional to the size of the players bid.*

can be manipulated to a player's advantage if the player is not honest about their valuations.

These problems are exacerbated by the prescriptive nature of the results the algorithms produce. Typically, algorithms produce a single allocation for the players to accept or reject. This does not provide the players with sufficient opportunity to consider alternatives, reconsider their valuations, or to engage in negotiation.

3. Metrics for Filtering Candidate Allocations

If each resource is indivisible, there are p^r possible allocations, where p is the number of players and r is the number of resources. Despite the exponential growth in the number of possibilities with the number of resources, the values of p and r are typically small, and generating all possible allocations in computer memory is not especially time-consuming. Once all possible allocations are generated, a visualization of a filtered set of possibilities can be browsed, even on a computer with modest performance. If the number of resources is large, the resources can be divided into sets, and the sets can be individually allocated.

The metrics that should be utilized in a visualization is a question for debate. We use the following metrics for each possible allocation with the acknowledgement that different problems might be better served with different measurements.

Subjective value of an item, expressed as a bid, is a measure of how much a particular player values an item. The subjective value of an allocation is the sum of the bids the player made on the resources they received.

We define the *market value* of an individual item as the value of the second highest bid for that item. When there are only two players, this is just the lower of the two bids. The sum of market values of all items provides a lower bound on the value of the whole set of resources. The sum of the market values of the resources a player receives in an allocation provides a lower bound on the value of that allocation.

In addition to their bids, we allow the players to specify *preference* scores to express their desire to win items independent of their bids. The sum of the preference scores in an allocation provides an additional measure for filtering.

A user applies filters to the displayed candidate allocations using sliders to specify minimum values for subjective, market, and preference values for each player. Additional filtering options allow the user to specify a set of resources that must be allocated to a specified player.

References

[BT96] Brams S. J., Taylor A. D.: *Fair Division: From Cake-cutting to Dispute Resolution*. Cambridge University Press, 1996. doi:10.1017/CBO9780511598975.001. 1

[CCG*22] Chaudhury B. R., Cheung Y. K., Garg J., Garg N., Hoefer M., Mehlhorn K.: Fair division of indivisible goods for a class of concave valuations. *Journal of Artificial Intelligence Research 74* (2022), 111–142. 1

[Mou19] Moulin H.: Fair division in the internet age. *Annual Review of Economics 11* (2019), 407–441. 1

[Rai82] Raiffa H.: *The Art and Science of Negotiation*. The Belknap Press of Harvard University Press, 1982. 1

[Wil19] Wilke C. O.: *Fundamentals of data visualization: a primer on making informative and compelling figures*. O'Reilly Media, 2019. 2

Proceedings published by Eurographics - The European Association for Computer Graphics.