

# Space Bundling for Continuous Parallel Coordinates

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## Abstract

*Continuous Parallel Coordinates (CPC) are a visualization technique used to perform multivariate analysis of different scalar fields defined on the same domain. While classic Parallel Coordinates draws a line for each sample point, a CPC visualization uses a density-based representation. An interesting possibility for the classic method is to highlight higher-dimensional clusters using edge bundling, where each line becomes a spline bent towards the centroid of the cluster. This often leads to expressive, illustrative visualizations. Unfortunately, bundling lines is not possible for CPC, as they are not involved in this method. In this paper, we propose a deformation of the visualization space for Continuous Parallel Coordinates that leads to similar results as those obtained through classic edge bundling. We achieve this by performing a curved-profile transformation in image space. The approach lends itself to a computationally lightweight GPU implementation. Furthermore, we provide intuitive interactions with the bundled clusters. We show several examples of our technique applied to a commonly available data set.*

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation—Bitmap and framebuffer operations Line and curve generation

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## 1. Introduction

Continuous Parallel Coordinates (CPC) are a visualization method introduced by Henrich et al. [HW09] used to perform multivariate analysis on data defined on a continuous domain. With this approach it is possible to visualize different scalar fields belonging to the same physical phenomenon. This allows the user to identify multidimensional trends otherwise difficult to spot.

CPC are based on parallel coordinates plots (PCP) [Weg90, ID90]. In PCP, an observation point is rendered as a polyline intersecting the parallel axes at a position proportional to its value. While the classic approach uses polylines to represent discrete samples, CPC provides a density-based continuous visualization of continuous data, defined via interpolation on a 2D or 3D grid. The density is computed based on Continuous Scatter Plots (CSP) [BW08] exploiting the point-line duality. Each pixel in the CPC accumulates the density of its dual line in the corresponding CSP.

The possibility to spot multidimensional clusters together with their trends is of high importance to acquiring a better understanding of the internal structure of multivariate data. In discrete parallel coordinates this challenge is addressed by a well known technique called *edge bundling*. In this approach each polyline between two axes is bent towards the center of the respective cluster. The resulting visualizations are usually intuitive and illustrative as they explicitly display multidimensional clusters directly in the geometry of the plot.

More illustrative visualizations of clusters are also needed in case

of CPC. Unfortunately, edge bundling can not be applied as CPC does not involve lines. In this paper we address this challenge by introducing a new space bundling approach for Continuous Parallel Coordinates which provides similar illustrative visualizations as the classic edge bundling technique. In our approach we visualize the density description of each multidimensional cluster between two axes as a different layer, and we deform it by vertically shifting each pixel of the visualization towards its center according to a curved profile. The result is an intuitive visualization which explicitly displays each multidimensional cluster directly in the shape of its density description. Our approach also allows the user to focus on different clusters by interactively highlighting them. We conclude our paper by comparing our method with an existing edge bundling approach and show several examples of our method applied to the Isabel data set used in the IEEE Visualization Contest 2004 [Isa].

The paper is organized as follows: we briefly review the related work in Section 2, we illustrate our approach in Section 3, we show examples of applications of our method in Section 4 and we conclude in Section 5.

## 2. Related Work

**Continuous Parallel Coordinates** CPC have many uses in visualizing scientific data. Grottel et al. [GHWG14] used CPC as coordinated views with temporal heat maps and Continuous Scatterplots (CSP) [BW08] in order to investigate multidimensional continuous trajectories such as robot arm movements or angles of joints in

human motion. The features of CPC have been extensively investigated by Lehmann and Theisel in [LT11]. Here the authors show the existence of feature curves in CPC that can be analyzed and classified to improve their resulting visualization. Moreover, they demonstrate the relation between such features and discontinuities in CSP.

**Edge Bundling Approach** Edge Bundling was proposed for the first time by Holten et al. [Hol06] to provide less cluttered visualizations for hierarchical data. McDonnell et al. [MM08] applied edge bundling to PCP. A prerequisite is that the data is clustered. Each polyline is replaced with a polycurve that is bent between two axes towards centroid of the corresponding cluster. While this approach reduces the visual clutter of the visualization, the adopted polycurves are  $C^0$ -continuous making them difficult to follow over the plot. To avoid this, Heinrich et al. [HLK\*11] adopted  $C^1$ -continuous polycurves. Palmas et al. [PBO\*14] propose an edge bundling approach based on density-based one dimensional clustering. Here they replace each polyline with a cubic Bézier spline and group them in order to render them as polygons.

Unfortunately, despite the fact that an explicit visualization of clusters in CPC would be of great use, until now no bundling approach has been proposed for this technique as CPC do not involve lines. In the next section we illustrate our new space bundling approach that deforms the original visualization of CPC providing results similar to original edge bundling.

### 3. Method

#### 3.1. Clusters

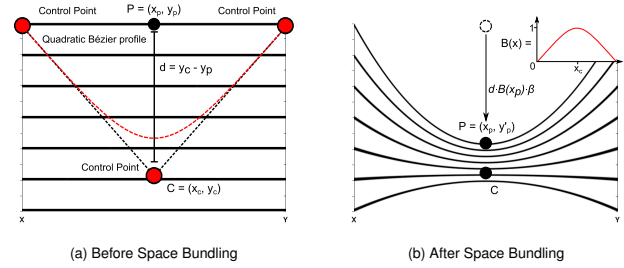
In the context of CPC, a segmentation of the continuous domain provides the analogon to clusters in the classic PCP. We assume such a segmentation to be given; it could be based on topology (Morse or Morse-Smale complex), thresholding, and many other methods.

We compute the density description of CPC by exploiting the point-line duality between CSP and CPC following the technique called *gathering approach* [HW09]. Given two consecutive axes in CPC, the dual CSP is defined by the two represented dimensions.

To perform our bundling technique we need to segment the density of the dual CSP according to each different cluster. We achieve this by representing each cluster as an additional scalar field called *cluster field*. A cluster field defines the inclusion of each point in its relative cluster by assigning them a continuous value between 0 (not included) and 1 (included). Note that the cluster fields are sampled on the same grid of the original ones and trilinear interpolation is used. Note that the sum of all cluster fields is a field with a constant value of 1. The values of these cluster fields are used to weight the density contribution of each point in all dual CSP defined in the current CPC. Each CSP is then used to compute the dual density in CPC of its respective cluster. These densities are finally rendered as different *cluster textures* on top of each other. Examples of results of this technique are shown in Figure 2 and 4.

#### 3.2. Space Bundling

Our approach aims to provide a more informative visualization of clusters in CPC by bending their original density description towards



**Figure 1:** Our space bundling performs a deformation of the original CPC visualization. Our method will move the point  $P$  towards the cluster center  $C$  according to the formula illustrated in Section 3.2. The red dashed curve shows the quadratic Bézier profile used to define the weighting function  $B(x)$ .

their center. This *space bundling* is done by applying a *vertical linear transformation* to every pixel of each cluster texture. As shown in Figure 1, this transformation is defined by a curved profile which defines the shape of the final bundled image. While any curve profile can be adopted, in this paper we will use a quadratic Bézier profile as this enables the comparison between our method and the original PCP bundling approach of McDonnell and Mueller [MM08].

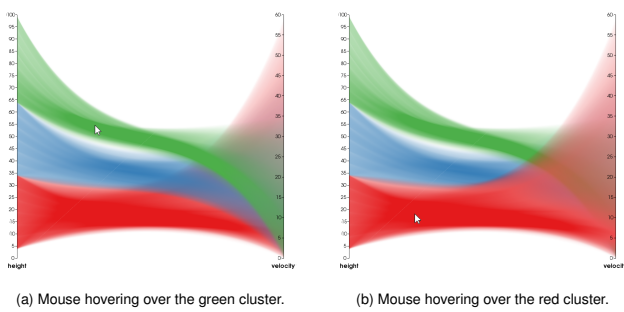
We define space bundling as follows. Given two consecutive axes in CPC, let  $C = (x_c, y_c)$ , the center of a cluster in screen coordinates computed as the average of the observation points in that cluster. For each pixel  $P = (x_p, y_p)$  in the cluster texture, we calculate  $y'_p$  which identifies the new vertical position of  $P$  in the bundled cluster texture. Such a position is defined as  $y'_p = y_p + d \cdot B(x_p) \cdot \beta$  where  $d = y_c - y_p$  is the vertical distance between  $C$  and  $P$ ,  $B(x_p)$  is a weighting function with values between 0 and 1 and  $\beta$  is an additional parameter which can be interactively modified by the user to vary the amount of bundling in the resulting image. In order to bundle the cluster texture toward  $C$ , the function  $B$  needs to have a curved profile and its maximum at  $x_c$ . In our case we defined our weighting function according to the profile of a quadratic Bézier curve with its first and third control points on the two sides of the cluster texture and its second control point on  $C$ . An example of such a profile is shown in red in Figure 1a. After having calculated  $y'_p$ , the density represented by  $P$  is then accumulated on the pixel having coordinate  $(x_p, y'_p)$ . If  $y'_p$  is not an integer coordinate the density of  $P$  is distributed between the involved pixels.

#### 3.3. User Interactions

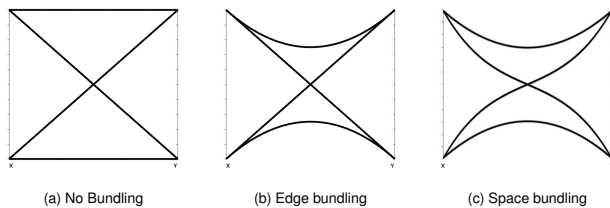
Our method allows the user to interactively explore the visualized bundled space as he is allowed to focus on a particular density by simply hovering on it with his mouse. In this way our visualization will put the selected cluster on top of the others allowing an easier determination its features. An example of this is shown in Figure 2. Here the users interact with the visualization to move the red cluster on top of the others while avoiding occlusion issues.

#### 3.4. Implementation

Our space bundling method is implemented in C++ using OpenGL and the VTK libraries [SML06]. In order to calculate the base



**Figure 2:** *Hovering on a cluster automatically moves it on top of the others in order to facilitate its visual inspection.*



**Figure 3:** *Comparison between classic parallel coordinates 3a, the edge bundling approach of McDonnell and Mueller [MM08] 3b, and our space bundling 3c. Edge bundling is performed per line while space bundling is performed per pixel resulting in the additional deformation of the diagonal lines in 3c.*

density description of CPC using CSP and the gathering approach from Heinrich et al. [HW09], we utilized the publicly available code from the Continuous Scatterplots project web page [CSP]. The vertical shifting computation took place entirely on GPU using a GLSL fragment shader. This shader takes as input the curved Bézier profile as a 1D floating point texture and produces the vertical shifting in parallel for each fragment. All density computations for each cluster and the vertical shifting were computed on a high-resolution floating point render target.

#### 4. Results

In this section we compare our space bundling with an existing edge bundling approach proposed by McDonnell and Mueller [MM08]. We then show and compare results obtained by visualizing a commonly available data set with our approach, discrete parallel coordinates and CPC.

**Comparison with edge bundling** In Figure 3 we show our space bundling approach and the edge bundling proposed by McDonnell and Mueller in [MM08] with respect to classic parallel coordinates. In order to have an equivalent transformation for the two bundling methods, in both cases we placed the bundling point in the center of the space between the two axes. While Figure 3 shows visual similarity between the two approaches, an important difference is the deformation of the diagonal lines in Figure 3b and in Figure 3c. This occurs in the edge bundling approach since a different

transformation is defined for each line, preserving in this case the shape of the oblique ones. In the space bundling approach we instead apply the same transformation to all the image space pixels, thus losing the notion of the single line.

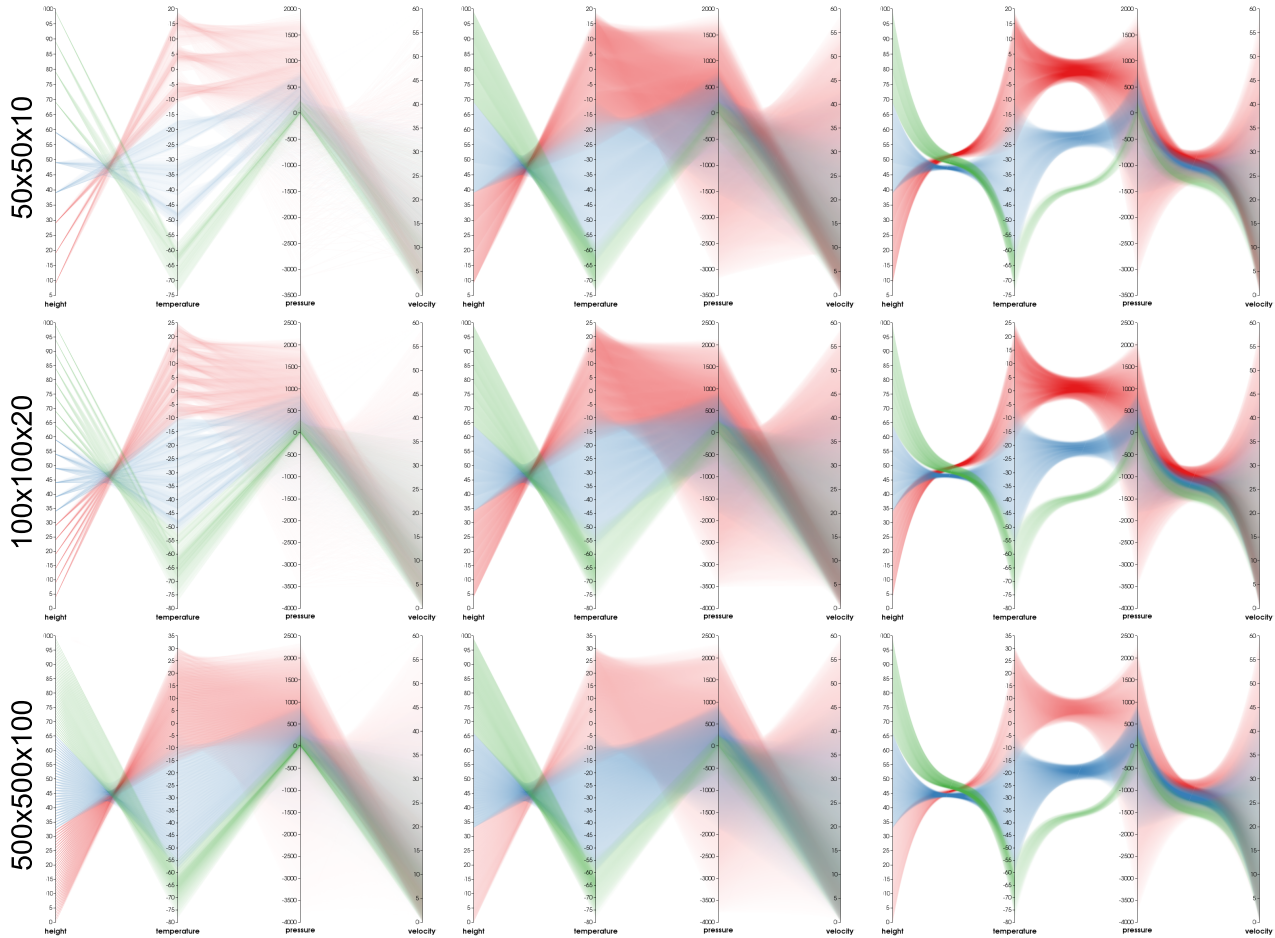
**Visualization of the Isabel dataset** Figure 4 shows a comparison between discrete density-based parallel coordinates, CPC and our bundled CPC in the visualization of the Isabel data set [Isa]. This data set was provided for the IEEE Visualization Contest 2004 and consists of 11 atmospheric attributes for 48 time steps with spatial resolution 500x500x100. For our results we visualized only the first time step. Discrete parallel coordinates were computed by counting how many lines were crossing each pixel in the visualization while CPC were computed using the *gathering approach*.

The four visualized dimensions are height, temperature, pressure and velocity. Temperature and pressure were already included in the data set. Height was computed according to the vertical spatial dimension and velocity was computed as the magnitude of the wind velocity. In our comparison we visualized the Isabel data set with three different spatial resolutions: 500x500x100, 100x100x20 and 50x50x10. All images in Figure 4 show a segmentation of the Isabel data set into three clusters representing low (red), medium (blue) and high height (green). As already stated by Heinrich et al. in [HW09], in the cases of low and medium resolution, discrete parallel coordinates introduce visualization artifacts. In fact in the top right sub-figure of Figure 4, they wrongly show the presence of many clusters in the height dimension conflicting with the real structure of the data. CPC does not present such a problem as it provides a correct representation of the continuous data at any resolution. Our space bundling method for CPC preserves this property and moreover shows the trend of each cluster more clearly. In fact while in normal CPC the density of a cluster can hide information due to occlusion, in our approach each cluster trend is shown explicitly by the deformation of the displayed density. An example of this is shown in Figure 4. While the main trend of the red cluster is not easily understandable in the normal CPC (especially in the third and the fourth axes), with our bundle approach the user can easily understand that the data contained in the red cluster generally has low height, high temperature, high pressure and low velocity.

#### 5. Conclusion and Future Work

In this paper we presented a space bundling approach for CPC which deforms the density description of each cluster by applying a vertical shifting transformation to each pixel in the image space following a curve profile. This provides an aggregated view of each clustered density similar to the results of edge bundling for discrete parallel coordinates. It helps the user to understand the internal features of the data in a more intuitive way as they are shown directly in the geometry of the plot. Moreover, we provide additional interactions allowing the user to focus a clustered density by hovering over it with his mouse.

The main limitation of our approach comes directly from the space deformation applied to CPC. In fact, transforming each pixel in CPC invalidates the dual relation between CPC and CSP. This is not a critical issue for our work since we want to provide an abstraction of the original visualization which does not depend on



**Figure 4:** Results of our method applied to 4 scalar fields of the Isabel data set. Here we segmented the data set in three different clusters: low height (red), medium height (blue) and high height (green). Each row shows such segmentation with discrete parallel coordinates, CPC and bundled CPC with different spatial resolutions: 50x50x10, 100x100x20 and 500x500x100. With low resolution, discrete parallel coordinates introduce visualization artifacts, while in CPC the structure of the data is preserved. Our bundled CPC both preserve the structure of the data and shows clearly the trends of every cluster due to the deformation of the density.

this relation and shows the main trend of the plot in an illustrative way.

In our work we consider clusters to be predefined additional scalar fields already present in the data set. A future extension for our method could be to interactively specify clusters directly on the visualization.

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