Isogeometric Analysis for Modelling and Design

A. Riffnaller-Schiefer†[1], U. H. Augsdörfer‡[1] and D. W. Fellner§ [1,2]

[1]Institut für ComputerGraphik & Wissensvisualisierung, TU Graz, Austria
[2]TU Darmstadt & Fraunhofer IGD, Germany

Abstract

We present an isogeometric design and analysis approach based on NURBS-compatible subdivision surfaces. The approach enables the description of watertight free-form surfaces of arbitrary degree, including conic sections and an accurate simulation and analysis based directly on the designed surface.

To explore the seamless integration of design and analysis provided by the isogeometric approach, we built a prototype software which combines free-form modelling tools with thin shell simulation tools to offer the designer a wide range of design and analysis instruments.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Physically based modeling

1. Introduction

One challenge in the computer aided design (CAD) of new products is to ensure structural stability of the resulting geometry. Finite Element (FE) analysis, typically based on a FE mesh derived from the CAD model, is used to test the stability of a virtual object. Isogeometric analysis (IGA) is a relatively new FE method, where analysis is based directly on the CAD model. IGA has been successfully applied using geometry representations commonly employed in CAD or animation, namely NURBS [HCB05], subdivision surfaces [COS00], and more recently T-splines [BCC*10]. Because the approach does not require the creation of a separate FE mesh, IGA facilitates an optimal integration of modelling and analysis tools into a single software.

We present an IGA framework based on NURBS compatible subdivision [CADS09]. This framework combines the advantages of NURBS and subdivision surfaces for analysis: Like with NURBS, arbitrary degrees offer greater flexibility for modelling and faster convergence for analysis. Also, rational representations, necessary for an accurate description of conic sections, are provided. Because non-uniform knot spacing is supported, creases and boundaries can now be parametrised correctly in a subdivision setting.

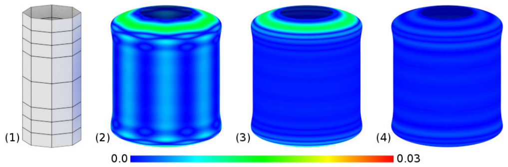

Figure 1: *Example of a pipe subject to internal pressure. A control mesh (1) is refined once to increase the degrees of freedom. Simulation results (2-4) are coloured based on error to the analytic solution. Non-rational CAD representations, like Catmull-Clark subdivision surfaces (2), lead to vertical artefacts in the analysis result because conic sections cannot be described accurately. These artefacts do not occur when employing a surface representation which includes a rational expression, like NURBS-compatible subdivision surfaces of degree 3 (3). Using higher degree surfaces, like degree 5 NURBS-compatible subdivision (4), improves the convergence of the simulation and enhances the results for the same number of elements in the analysis.*

The arbitrary topology property of subdivision surfaces provides the designer with watertight free-form meshes, an important prerequisite for correct analysis. The multilevel resolution property of subdivision surfaces enables the designer to easily perform the analysis at different resolution levels, depending on the required accuracy of the simulation results.

---

† a.schiefer@cgv.tugraz.at
‡ u.augsdorfer@cgv.tugraz.at
§ d.fellner@cgv.tugraz.at

DOI: 10.2312/egsh.20151004

We propose to include an IGA framework based on NURBS compatible subdivision [CADS09] into modelling software to give designers access to analysis and simulation tools throughout all design stages within their software. By providing a general IGA framework, many new design tools may be integrated into modelling applications.

Some of the advantages of having analysis tools available during the design stage are described in Section 3. In Section 4 we show how we provide physics based modelling tools without difficulty within our IGA design and analysis software to offer a clay-like modelling paradigm in addition to design by modification of control points.

2. Simulation

The IGA implementation presented here focuses on isogeometric thin shell analysis. A thin shell is a 3D elastic solid of which one dimension is small with respect to the two others. This particular geometry covers a wide range of engineering designs. For analysis this geometry is formulated in terms of the middle surface of the shell and we only require two-manifold meshes for the simulation. This corresponds directly to how such objects are represented in CAD modelling applications.

The mechanical response of a thin shell to forces is described by the Kirchhoff-Love theory in terms of the first and second fundamental forms of the original and deformed surfaces (e.g., [TWK59]). The response of a thin shell to external forces is particularly sensitive to the shape of a shell surface. Figure 1(2) shows the effect of small deviations from a true conic section due to the CAD representation not supporting rational expressions.

To provide the important rational description and to be able to explore improved convergence of higher degree surfaces, see Figure 1(4), we extended the subdivision based thin shell analysis formulation by Cirak et al. [COS00] to Cashman's NURBS compatible subdivision [CADS09]. In particular this required an extension of Stam's exact evaluation algorithm [Sta98] to rational, higher degree surfaces. Further details can be found in [RSAF15].

3. Analysis Tools

Thin shell analysis not only computes deformations due to external forces, but also includes the calculation of important physical quantities like internal material stresses caused by the deformation. Based on the stresses and the assumed material used for the simulation the structural stability of thin shells can be assessed. We integrated IGA tools into modelling software to provide the designer with valuable feedback on structural properties of the design throughout every design stage.

The von Mises stress is a scalar value that can be compared to the yield strength of a material to predict structural problems. We compute von Mises stresses at material points on the middle surface of the thin shell, described by the subdivision surface, to get a scalar field which can be visualised directly in the modelling application by colouring the surface, see Figure 2(3). The maximum value of the colour ramp used to visualise the stresses is set to the yield strength of the material chosen by the designer. This visualisation enables the designer to detect structural problems in the design for a particular choice of material. The designer may change shape or material properties accordingly to improve the design with respect to structural stability.

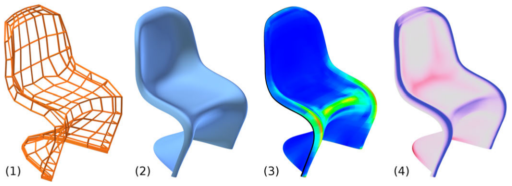

Figure 2: *The subdivision control mesh of a chair (1) used for design and analysis and the limit CAD surface of the chair model (2). The surface is coloured according to the von Mises stress (3) computed from the deformation caused by a force "sitting" on the chair. The force is defined by the designer by placing a proxy force as shown in Figure 5. The visualisation enables the designer to detect structural problems with the design. The framework also simplifies the evaluation of other surface quantities, e.g. mean curvature (4).*

Having IGA simulation available as a general tool within a modelling application offers many new possibilities to analyse the designed surfaces.

Being able to integrate arbitrary functions on the designed surface, a requirement for isogeometric analysis, simplifies the implementation of many common mesh analysis features in design software. For example, a common task in CAD is to compute the surface area of the designed free-form surface. While almost all NURBS based engineering/CAD software have this feature available, most free-from modelling tools based on subdivision do not. At best, the surface area of the subdivision surface is approximated with the surface area of a subdivided mesh. With the isogeometric analysis framework available the surface area of the subdivision limit surface can be computed without having to create a dense polygonal mesh. Using the IGA framework we can compute the surface area of the limit surface with high accuracy with just a few lines of code provided by the user or a plugin:

```
def element_area(shape_fun, coords, weight,
                 element_accumulator):
  # compute tangent vectors
  t = shape_fun.first_derivatives * coords
  # area is norm of normal vector
  a = norm(cross(t[0], t[1]))
  return element_accumulator + a * weight

# integrate area at element quadrature points
elem_results = surf.integrate(element_area)
```

```
total_area = sum(elem_results)
```

The ability to evaluate and integrate arbitrary functions can also be used to efficiently compute various other surface properties requiring derivative information (e.g. curvature, see Figure 2(4)), or volume information which considers the assumed thickness of a shell like structure.

4. Modelling Tools

By integrating isogeometric thin shell simulation into the modelling software new interesting modelling tools can be provided.

Because the simulation models realistic physical behaviour of thin objects, it can be used to provide the designer with a range of physics based modelling tools in addition to simply moving control points. Physics based modelling tools are particularly interesting for designs which are described by dense control meshes where many control points may have to be moved to achieve a new variation of the design (see [BS08] for an overview).

The geometry of a thin shell shape may be modified by either defining forces acting on the thin shell which cause deformations, much like modelling virtual clay, or by setting constraints. Typically, constraints and forces are used together to define deformations. Constraints are used to fix parts of the surface in place while forces are applied to other parts of the surface, causing the deformation.

Forces can also be defined to specify interesting new modelling operations. For example, using the thin shell simulation, inflation can be defined in terms of a uniform internal pressure (see Figure 3(4)). Additionally, constraints can be used to restrict the inflation to some parts of the mesh.

By modifying/displacing control points and constraining these control points to their new position the thin shell simulation will determine a new geometric shape depending on the chosen constraints and predefined material properties. In this way the designer can modify the geometry of the subdivision surface by applying a few displacement constraints to the subdivision control points to deform the resulting subdivision limit surface in a physically realistic way. In the simplest form, these constraints can be specified by the user as (x,y,z) displacements for certain control points of the subdivision surface. For example, a prescribed displacement of $(0,0,0)$ fixes the vertex in place, while all non constraint vertices are free to be moved by the simulation.

Figure 3 compares a modification of a design by moving control points (2) to physics based modelling tools (3, 4). Both modelling modes are readily available to the designer from within one design software.

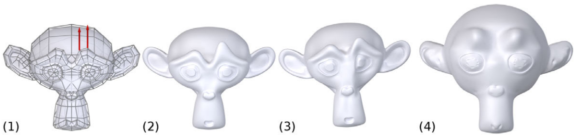

Figure 3: *(1) A subdivision control mesh together with its corresponding limit surface. (2) Limit subdivision surface after translating two control points as indicated by red arrows in (1). (3) Limit thin shell surface after constraining control points to new positions, as indicated by red arrows in (1). (4) Inflated thin shell surface as a result of applying internal pressure to middle surface described by control mesh shown in (1).*

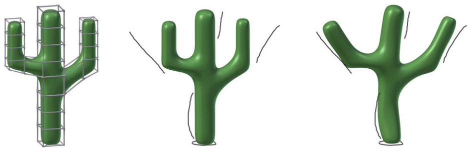

Figure 4: *Constraints specified by user sketched curves. To deform a subdivision surface defined by a control mesh (left) the user sketches the outline of the deformation (center) and the surface deforms due to constraints created from the sketched curves (right).*

Intuitive methods to specify constraints on control points may be implemented. Figure 4 shows an example of a sketch based deformation approach. To constrain control points of the surface to the sketching curves, the curves are first discretised into a number of line segments. Then, for each vertex in the discretised curves, the closest control point p_i of the subdivision surface is searched. Each curve vertex is added to a set of potential constraint-positions T_i for this closest control point. Afterwards, an offset vector is computed from all control points p_i to the nearest curve vertex in T_i. This offset vector is then applied as a displacement constraint for the simulation. Alternatively, constraints may be defined by selecting and dragging parts of the subdivision surface. Control points are selected by mouse click, while the drag defines the displacement offset that is applied to the selected control point.

More complex constraints like pinning of material points, so that the thin shell surface passes through a given point, can be achieved with Lagrange multipliers, which allow to express constraints as linear combinations of vertex displacements. To pin a material point of the thin shell, the linear combination of vertices of the subdivision mesh defining the limit point location, is constraint to zero.

5. Implementation

We implemented a prototype based on the ideas presented in the previous sections for the open source modelling software Blender.

The implementation consists of two parts. The first part

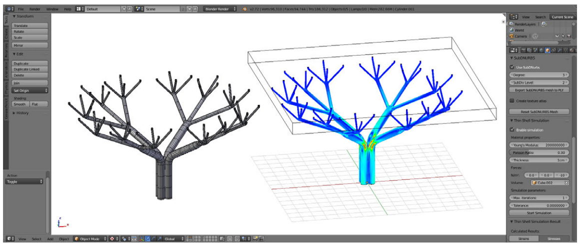

Figure 5: *Visualisation of von Mises stresses on a roof supporting structure subject to a load. The load is defined by the designer within the modelling software Blender by placing proxy geometry on the top of the structure.*

adds support for NURBS compatible subdivision surfaces to Blender. This geometry representation is currently not available in any other modelling software. The second part provides the isogeometric thin shell framework based on the NURBS compatible subdivision surfaces. Both parts are implemented as a plugin for Blender using its Python API. The thin shell simulation can be used in two modes from within Blender. In the first mode, the user specifies forces and constraints and then explicitly starts the simulation. This is useful if the user wants to setup a particular scenario and then use the deformed result or inspect the resulting stresses, as shown in Figure 5. The second mode enables the user to interactively deform the surface. Once activated, the user can interactively click and drag the surface, or sketch deformation outlines as described in Section 4. Stress visualisation is also available after interactive deformation of the surface.

To optimise the response time of the simulation in the interactive mode, the stiffness matrix required for the simulation is computed before the user interacts with the surface, for example while the designer specifies additional forces and constraints. The stiffness matrix can then be used to quickly compute linear deformation solutions for any user interaction, as long as the initial geometry does not change. If the geometry of the surface changes, the stiffness matrix is recalculated.

The interactive mode is very useful to quickly create realistic deformations for a coarse subdivision surface. The resolution of the subdivision surface can be chosen according to the desired detail of the required simulation or physical modelling. However, the time of analysis increases with the density of the mesh.

6. Discussion

We have presented an implementation of an IGA framework based on NURBS compatible subdivision. This framework was employed to analyse thin shells geometries which are notoriously sensitive to small shape deviations. The presented framework offers the advantages of both, NURBS and subdivision for analysis. It supports rational expressions, necessary for the correct description of conic sections, and higher degrees, which have shown to improve the convergence of analysis. Subdivision surfaces offer watertight meshes, a prerequisite for analysis and multiresolution capabilities.

The IGA framework has been incorporated in a modelling software. We have shown that isogeometric analysis in design applications is interesting with respect to simplifying the implementation of current mesh analysis features in the software as well as for adding new features. Users of the software get valuable feedback on the physical plausibility of their design as well as more intuitive modelling tools for creating realistic deformations. If the isogeometric analysis framework is exposed to the scripting API of the modelling application, many new types of plugins may be created.

The biggest challenge is currently the performance of the simulation. For complex meshes the computation of the simulation result can still take a long time. However, as for example the work by Weber [WBS*13] shows, near real time performance can be achieved for FE computations by offloading work to the GPU. This is a very promising direction for future optimisations of the interactive implementation of IGA tools.

References

[BCC*10] BAZILEVS Y., CALO V., COTTRELL J., EVANS J., HUGHES T., LIPTON S., SCOTT M., SEDERBERG T.: Isogeometric analysis using T-splines. *Computer Methods in Applied Mechanics and Engineering 199* (2010), 229–263. 1

[BS08] BOTSCH M., SORKINE O.: On linear variational surface deformation methods. *IEEE Transactions on Visualization and Computer Graphics 14*, 1 (Jan. 2008), 213–230. 3

[CADS09] CASHMAN T., AUGSDÖRFER U., DODGSON N., SABIN M.: NURBS with Extraordinary Points: High-degree, Non-uniform, Rational Subdivision Schemes. *ACM Transactions on Graphics 28*, 3 (2009). 1, 2

[COS00] CIRAK F., ORTIZ M., SCHRÖDER P.: Subdivision surfaces: A new paradigm for thin-shell finite element analysis. *Int. J. Numer. Meth. Eng. 47*, 12 (2000), 2039–2072. 1, 2

[HCB05] HUGHES T., COTTRELL J., BAZILEVS Y.: Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Comp. Methods Appl. Mech. Engrg. 194* (2005), 4135–4195. 1

[RSAF15] RIFFNALLER-SCHIEFER A., AUGSDÖRFER U., FELLNER D.: Isogeometric shell analysis with NURBS compatible subdivision surfaces. *submitted to Applied Mathematics and Computation* (2015). 2

[Sta98] STAM J.: Exact evaluation of catmull-clark subdivision surfaces at arbitrary parameter values. In *Proceedings of SIGGRAPH 1998* (1998), pp. 395–404. 2

[TWK59] TIMOSHENKO S., WOINOWSKY-KRIEGER S.: *Theory of plates and shells*. Engineering societies monographs. McGraw-Hill, 1959. 2

[WBS*13] WEBER D., BENDER J., SCHNOES M., STORK A., FELLNER D.: Efficient gpu data structures and methods to solve sparse linear systems in dynamics applications. *Computer Graphics Forum 32*, 1 (2013), 16–26. 4