A preliminary analysis of methods for curvature estimation on surfaces with local reliefs

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Abstract

Curvature estimation is very popular in geometry processing for the analysis of local surface variations. Despite the large number of methods, no quantitative nor qualitative studies have been conducted for a comparative analysis of the different algorithms on surfaces with small geometric variations, such as chiselled or relief surfaces. In this work we compare eight curvature estimation methods that are commonly adopted by the computer graphics community on a number of triangle meshes derived from scans of surfaces with local reliefs.

CCS Concepts

• ***Computing methodologies*** → ***Shape modeling; Shape analysis;***

1. Introduction

Surface curvature yields several insights on the local shape geometry, such as convexity/concavity, unfoldability and reflecting lines, being consistent with the human perception of a geometric shape [PT96]. For this reason, many shape processing and analysis algorithms [Gol05,DC16] ground on surface curvatures. A detailed introduction of curvatures on surfaces can be found in [Lip69]. Briefly, the curvature of a surface S (at least C^2-smooth) in one of its points P is a numerical estimation of the bending of S in P. Curvatures and their derived entities, such as the mean curvature, the Gaussian curvature, the Shape Index and the *Curvedness* [KvD92], are widely used to describe the local geometric properties.

While the definition of curvatures is not ambiguous for smooth surfaces, it is not well defined for discrete representations of surfaces. Common computational strategies are the evaluation of the curvatures for a surface that locally approximates the mesh or the definition of discrete curvature tensors. The literature on curvature estimation is vast and we cannot do justice to it here. For details, we refer to the comparative studies on methods for estimating curvatures on triangle meshes [GG06, MSR07] and the recent benchmark [VVP*16]. As a broad classification, we classify the methods as: *fitting* methods, based on the fitting of mathematical surface primitives (for instance quadratic surfaces like spheres, or cubic Bézier patches or Hermite RBF fitting) [GI04, CP03, GG07, GGG08, PV18]; *direct discretization* methods of the curvature in a vertex (e.g., in terms of the angle excess or variations [MPS*04]) and of the curvature tensor [MDSB03, Tau95, Rus04, KSNS07, DW05, ZGYL11]; *indirect approximations* of related quantities (e.g. second form) [CSM03, CSM06, LBS07].

A limitation of the existing benchmarks for the comparison of curvature estimators is that the curvatures are evaluated on almost smooth surfaces and looking at the overall surface without focusing on the feature details. On the contrary, in this paper we focus on the practical behaviour of the algorithms on *local geometric variations*, such as chiselling, incisions, small bumps on the surface, to assess the capability of different approaches to identify of local features. Differently from [MSR07] and [VVP*16], the models we are considering do not correspond to smooth surfaces and are all obtained with scans of real objects, thus we do not possess the exact curvature values to compare the estimations obtained. For this reason, our comparison is mainly visual and quantitative metrics are provided only for the localization of curvature values on specific features and the frequency of the outlier values.

2. Experimental settings

We selected seven 3D models that we think are significant when looking for the characterization of local features and details. Figure 1 overviews the selected models. To represent these surface we adopt a triangle mesh, which is a standard-de-facto representation of models reconstructed from laser scans and for which there is a rich variety of algorithms. All surfaces are sampled with 1M vertices, with the exception of *M1* and *M7*, which have $\sim 550K$ vertices. Figure 1 visually represents the vertex density for each model; the red regions contain 20000 vertices, approximately. The local variations in these surfaces come in various shapes and degrees, ranging from sharp variations (facial traits of *M1* and *M4*) to smooth ones (hairs of *M3* or scales of *M2*), from small ones (like the hair of *M1*, *M5* and *M6*) to bigger ones (scales of *M2*), from frequently repeated (circlets of *M5* and *M6*) to more localized ones (door decorations of *M7*).

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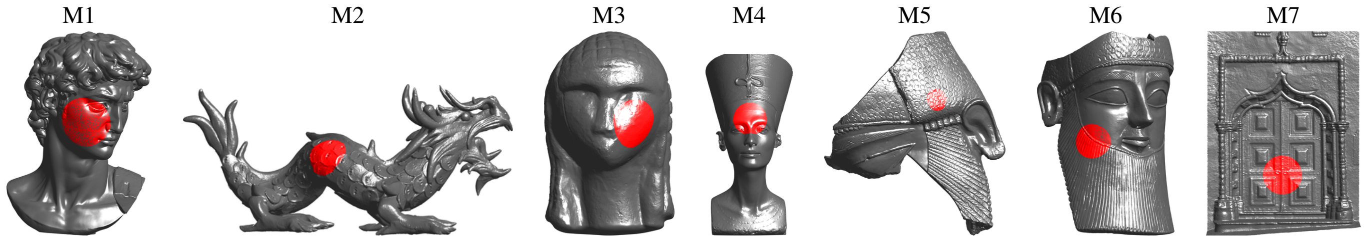

Figure 1: *Our test-beds.* Models 5 & 6 *come from the GRAVITATE use case [EU 18]. The red areas contain* 20000*, approximately.*

We selected eight representatives of the various curvature estimator strategies. Far from being exhaustive, our selection falls on implementations that are freely available and, in our knowledge, of common use in the geometry processing community. Namely, we are considering:

- the Algebraic Point Set Surface (AP) fitting method [GG07, GGG08] as implemented in [CCC*08];
- the curvature discretization based on the *cotan* discretization of the Laplace-Beltrami operator (MA) [MDSB03] following the implementation provided in [CCC*08];
- the pseudo-inverse quadratic fitting method (QF) in [CCC*08];
- the normal cycles approach (NC) [CSM03] as in [Pey];
- the curvature estimation based on an adaptive re-weighting of the vertices in the neighborhood proposed in [KSNS07] (KA) (authors' implementation);
- the discrete estimation of the second fundamental form proposed in [Rus04] (TR) (author's implementation);
- the normal curvature estimation based on the Euler formula proposed in [DW05] (DO) (authors' implementation);
- the least-squares curvature tensor approximation and iterative diffusion smoothing proposed in [Chu01] (CH) (authors' implementation).

For every method that allows the user to tune its parameters (namely NC, KA, AP, MA) we adopt the default algorithm settings as proposed by the authors without altering the neighbour size or the smoothing intensity. While this is in itself a fair way for comparing different methods, there could be a parameter optimization that better suits the problem we are considering. Anyway, the lacks of criteria for tuning the parameters and the lack of ground-truths convinced us to keep the default settings. Depending on the technique, different surface variations are highlighted: this implies there is not a best method for all applications. Not only, the peculiarity of a method like the sensitivity to small scales of the geometric variations, could become a detriment in applications that need, for example, a noise estimation.

3. Results

In this paper we limit the comparison to the *mean curvature* values. Figure 2 visually overviews the results we obtained. The mean curvature is represented with colors and it ranges from -2 (blue) to 2 (red). Such an interval intuitively spans the visible curvature variations in the 3D models (i.e.: a rough approximation of the theoretical extreme values of the curvature). Curvature values that exceeds from that interval are considered as *outliers*. An exception

	M1	M2	M3	M4	M5	M6	M7
AP	5.5%	17.1%	8.3%	1.6%	54.7%	51.3%	76.8%
CH	>0.05%	0.9%	1.0%	>0.05%	10.3%	10.5%	35.1%
DO	0.1%	2.6%	0.7%	0.1%	28.2%	26.6%	52.4%
KA	>0.05%	1.1%	>0.05%	>0.05%	26.4%	24.3%	49.0%
MA	0.8%	5.8%	4.7%	0.3%	31.2%	29.4%	60.7%
QF	0.4%	2.3%	0.5%	>0.05%	24.7%	23.7%	49.3%
NC	0.7%	2.4%	0.1%	>0.05%	>0.05%	0.1%	0.2%
TR	0.1%	2.0%	0.3%	>0.05%	26.2%	24.6%	51.1%

Table 1: *Percentage of mesh vertices classified as a outliers.*

is the NC method, which approximates the tangent bundle of the curvature tensor rather than the curvature values. Looking at the curvature variation we select the interval $[-0.05, 0.05]$ as reference interval for NC.

As already highlighted in [VVP*16] every approximation algorithm suffers of ambiguities, like the non-uniqueness of the fitting surface or the sensitiveness of the method to local perturbations. Looking at Figure 2, we can see that DO, CH, KA, MA, QF and TR output very similar mean curvature estimations. This group of methods is able to effectively highlight small repeated variations (beard and circlets in *M5* and *M6*), while consistently keeping at 0 the curvature estimation on the flat areas. A downside is that these methods seem to output mainly extreme curvature values (around ± 2) and 0, rarely passing through other values (it can be observed by the lack of orange/light blue vertices). DO (see *M2*) presents a greater continuity of the colors. In this sense, the best approximation is obtained by NC (especially visible in *M1* and *M2*). Also, NC is one of the few methods that put high contrast in the curvature values on convex/concave areas of the model, together with AP. A downside of AP is its sensitiveness to really small variations, especially visible in the flat areas in *M7*.

Tables 1 and 2 present some statistics on the outlier distribution. For this analysis, the absolute value of the curvature estimations is considered. The values in Table 1 represent the percentage of vertices that are out of the expected curvature interval. The values in Table 2 give an idea of the range of variation of the mean curvature in correspondence of the outliers (in terms of the $5-th$ decil, last permil and maximum value). The results in Table 2 suggest that for meshes with dense vertices like those adopted in our experiments, the NC approach is the most stable in terms of the absolute variation of the mean curvature. Considering both Tables 1 and 2, the $5-th$ decil, which is an approximation of the average of the mean curvature in the outliers, is very close to 2 in most cases. Note that our definition of the curvature outlier bases on the empirical obser-

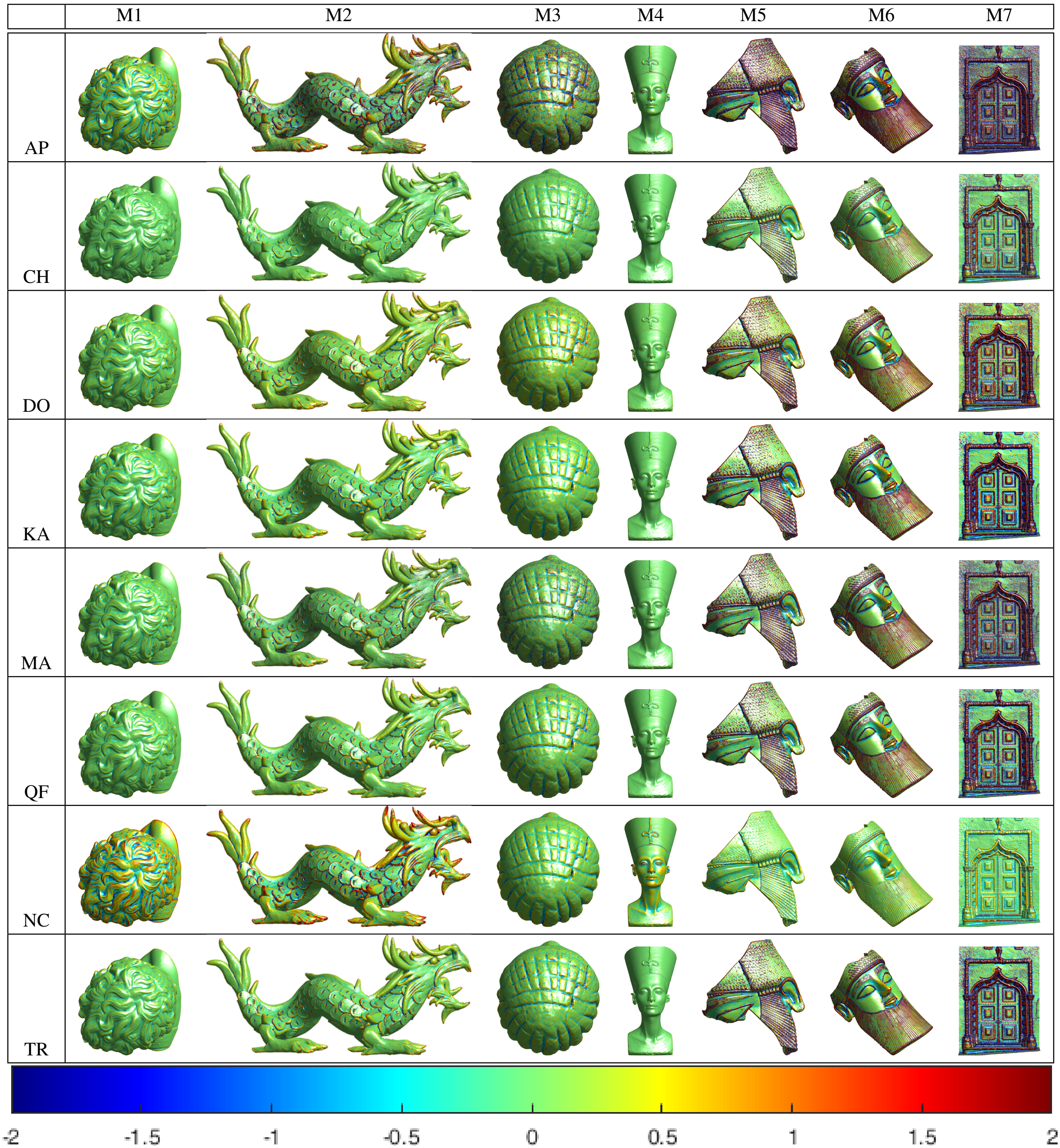

Figure 2: *Visual representation of the mean curvature values for the eight algorithms. The color-bar is reported at the bottom. The value for NC are re-scaled into* $[-2,2]$ *from* $[-0.05,0.05]$.

vation that for many smooth surfaces the mean curvature values are enclosed in the interval $[-2,2]$; however, the presence of many outliers does not necessarily imply that a method is unstable. Also, as in the case of NC, the values estimated by a given method could be consistent in terms of variation even if the values are not those that we expect from the theory behind the curvature on surfaces. This

	M1	M2	M3	M4	M5	M6	M7
AP	2.56 [9.28] <10^5>	2.98 [15.73] <10^6>	2.71 [11.74] <22.58>	2.47 [8.19] <11.55>	4.61 [30.53] <77.27>	4.77 [31.66] <87.44>	8.10 [72.47] <363.7>
CH	2.30 [14.04] <14.04>	2.50 [14.61] <39.77>	2.70 [13.65] <36.71>	2.34 [6.30] <6.30>	2.66 [12.18] <112.9>	2.76 [12.21] <185.1>	3.56 [25.53] <2649>
DO	2.24 [31.08] <31.94>	2.44 [10.45] <22.73>	2.47 [37.97] <187.1>	2.31 [8.88] <8.92>	3.28 [22.31] <164.1>	3.33 [20.86] <139.2>	4.76 [35.11] <582.4>
KA	2.27 [55.71] <55.71>	2.36 [7.49] <17.42>	2.33 [40.09] <40.09>	2.33 [6.95] <6.95>	3.17 [12.92] <78.18>	3.20 [12.73] <56.50>	4.38 [27.48] <5296>
MA	2.34 [15.89] <84.21>	2.66 [14.65] <26.51>	2.99 [38.48] <1097>	2.40 [12.20] <23.34>	3.38 [93.92] <10^8>	3.47 [130.8] <10^7>	5.06 [119.1] <10^5>
QF	2.34 [23.67] <32.23>	2.57 [22.70] <74.22>	2.39 [8.90] <182.9>	2.31 [7.30] <7.30>	3.24 [17.15] <1074>	3.27 [18.37] <6465>	4.44 [37.53] <9568>
NC	0.056 [0.246] <0.260>	0.056 [0.459] <0.821>	0.056 [0.112] <0.115>	0.052 [0.060] <0.060>	0.054 [0.092] <0.092>	0.057 [0.213] <0.213>	0.064 [0.303] <0.337>
TR	2.22 [12.93] <12.93>	2.44 [8.32] <14.23>	2.35 [14.84] <42.34>	2.24 [6.61] <6.61>	3.18 [14.29] <29.34>	3.23 [14.49] <43.14>	4.61 [34.26] <6485>

Table 2: *Approximated $5-th$ decil, [last permil] and <maximum values> of the curvature in correspondence of the outliers.*

fact confirms that all the methods provide a reasonable estimation of the curvature values, with the exception of NC, which captures the curvature variations rather its values.

4. Conclusion

The results shows that no a single estimator is suitable for all possible input data but the methods that smooth the curvature estimation in a vertex neighbour provide visually and quantitative better performances when highlighting features on scans of manufacts.

Further investigations are necessary for meshes with different densities of the vertex distribution because this would further alter the uniformity of the curvature estimation. Also, despite what stated in Section 2, different choices of the algorithm settings are worth to be further analyzed. Aside the mean curvature, we plan to extend the same analysis framework to other curvature-based properties like shape index and curvedness, both to confirm the behaviour of the algorithms for these quantities and to study which of these quantities is the most suitable for relief characterization.

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