

# Extending the Immediate Buckling Model to Triangular Meshes for Simulating Complex Clothes

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## Abstract

The immediate buckling model is an essential element for simulating realistic cloth animations without introducing buckling instability. The original model is restricted to structured regular quad meshes, by which its use is severely limited. This paper extends the immediate buckling model from its original formulation in terms of regular quad meshes to irregular triangular meshes, thereby significantly increasing the applicability of the technique. Using a model that included cloth-specific buckling and anisotropy, we produced realistic animations of quite complex clothes.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and RealismAnimation; I.6.5 [Simulation and Modelling]: Model Development

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## 1. Introduction

Thanks to the pioneering work of various groups over the past decade [4, 2, 14, 7, 1, 5, 3](#), cloth can now be simulated with remarkable realism. For example, natural wrinkles can now be produced using the particle model with the immediate buckling assumption, and the robustness of the collision handling in cloth simulations has been considerably improved. Along with the improvements that have been made in animation quality, the overall simulation algorithm has been refined such that it runs at a reasonable speed.

Given the progress in cloth simulation, one might wonder whether this technique could be used to create a fashion show, thereby avoiding the need to pattern, cut and sew real clothes, and to hire fashion models. The significance of attempting to simulate a fashion show is that it raises practically important issues. To produce an animated fashion show of acceptable quality, the designer should be allowed to use patterns of arbitrary 2D shapes, and the underlying physical model should possess the most fundamental properties of cloth, namely *cloth-specific buckling* and *anisotropy*.

The cloth-specific buckling proposed by Choi and Ko<sup>5</sup> increased the realism of cloth simulation remarkably. However, their model defined on regular quad meshes significantly limits the applicability of the technique, since the patterns can take on arbitrary 2D shapes. An obvious direction

for improving their technique would be to extend it to irregular triangular meshes.<sup>†</sup> This paper presents a solution to adapt the original model to work with irregular, unstructured triangular meshes.

The rest of this paper is organized as follows. Section 2 provides a survey of related cloth simulation studies; Section 3 describes our immediate buckling triangular mesh model; Section 4 presents the results of experiments; and finally, Section 5 concludes the paper.

## 2. Related Work

In computer animation, Terzopoulos et al.<sup>11</sup> were the first to develop a physical model for use in the simulation of cloth, and Carignan et al.<sup>4</sup> refined this model by adding damping and collision handling features. Breen et al.<sup>2</sup> developed a non-continuum particle model for predicting cloth drape. Later, Eberhardt et al.<sup>7</sup> adapted the model of Breen et al. to create a dynamic simulation method based on a Lagrangian formulation. Volino et al.<sup>14</sup> developed a cloth model based on elasticity theory and used a Newtonian formulation instead of a Lagrangian formulation. Eischen et al.<sup>8</sup> modelled

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<sup>†</sup> Triangular meshes have commonly been employed as the underlying topology of physical models of cloth<sup>1, 14</sup>. However, those models were not based on the immediate buckling assumption.

cloth using nonlinear shell theory. They used a standard nonlinear finite element procedure to obtain the system equation.

Another vital step in obtaining cloth motion is the numerical simulation of the underlying physical model. Since Baraff and Witkin<sup>1</sup> started using the semi-implicit method, the technique has become a popular technique for integrating the equations of motion in cloth simulation. A rigorous analysis of this method has been carried out by Volino et al.<sup>12,13</sup> and Hauth et al.<sup>9</sup>. Desbrun et al.<sup>6</sup> pre-computed the inverse of the simplified hessian matrix to make possible real-time cloth simulation. Kang et al.<sup>10</sup> proposed another variation of the semi-implicit method that avoids both solving the large linear system and pre-computing the inverted hessian matrix.

Choi and Ko<sup>5</sup> revealed that the previous physical models of cloth can cause the post-buckling instability, which can be problematic when wrinkles are formed. Such instability is independent of the numerical method employed because it is an inherent physical instability. Noting that the buckling behavior of cloth differs from that of other thin materials, they assumed compressive force on cloth immediately initiates buckling rather than compression. Simulations incorporating this *immediate buckling assumption* produced realistic wrinkles.

### 3. Immediate Buckling Model Extended to Triangular Meshes

In this section, we develop the triangular mesh version of the immediate buckling model by specifying how the stretch, shear, and bending interactions are modeled.

#### 3.1. Stretch and Shear Interaction Models

The physical model of cloth is largely determined by the energy functions that give the amount of internal energy contained in stretch, shear, or bending deformations. In this section, we define the energy functions for the stretch and shear interaction models, and derive the forces acting on the particles as a result of such deformations.

First, we define the stretch and shear energies on each triangle.<sup>‡</sup> Since clothing is constructed from pieces of cloth that are cut from a flat sheet, stretch and shear deformations are closely related to the parametrization  $S(u, v)$  that maps from the 2D parameter space to the 3D space. Suppose the triangle under consideration is composed of three particles. Let their parameter space coordinates be  $w_i = [u_i \ v_i]^T$ ,  $w_j = [u_j \ v_j]^T$  and  $w_k = [u_k \ v_k]^T$ , and let the corresponding 3D-space coordinates be  $x_i$ ,  $x_j$  and  $x_k$ , respectively. If the parametrization is approximated as a linear function for each triangle, the stretch model reduces to the one described

<sup>‡</sup> In quad meshes, the stretch and shear energies are defined on each edge.

in <sup>1</sup>.<sup>§</sup> Therefore the partial derivatives  $S_u = \frac{\partial S}{\partial u}$  and  $S_v = \frac{\partial S}{\partial v}$  are given by

$$\begin{bmatrix} S_u & S_v \end{bmatrix} = \begin{bmatrix} x_j - x_i & x_k - x_i \end{bmatrix} \begin{bmatrix} u_j - u_i & u_k - u_i \\ v_j - v_i & v_k - v_i \end{bmatrix}^{-1}, \quad (1)$$

and we can define the stretch energy as

$$E_{ST} = \frac{1}{2}A\{k_u(|S_u| - 1)^2 + k_v(|S_v| - 1)^2\}, \quad (2)$$

where  $A$  is the area of the triangle, and  $k_u$  and  $k_v$  are the stretch stiffness constants along the  $u$  and  $v$  directions, respectively.

In defining the shear energy function, however, we take an approach that is somewhat different from the methods employed previously. Most previous methods measure the shear deformation based on the angle between  $S_u$  and  $S_v$ <sup>1,14</sup>, or between the weft and warp directions of threads<sup>2</sup>, so that the deformation energy increases as the angle deviates from 90 degrees. However, this definition may cause *shear buckling* because the shear deformation is accompanied by local compression.

To avoid the shear buckling instability, we decomposed the shear deformation into *shear compression* and *shear stretch*. The immediate buckling assumption applies to shear compression: the compression arising from shear deformation is assumed to cause bending and thus is handled by the bending interaction model (Section 3.2). Therefore, the model does not give rise to post-buckling instabilities.

The shear stretch can be measured from the extensional strains in the diagonal directions. Under this definition, the equation for shear stretch energy will take basically the same form as Equation 2, except that the extension is measured along the diagonal directions  $\tilde{u}$  and  $\tilde{v}$ . If  $u$  and  $v$  are the unit vectors along the  $u$  and  $v$  directions in the parameter space, the shear stretch energy can be written as<sup>¶</sup>

$$E_{SH} = \frac{1}{2}A\{k_{\tilde{u}}(|S_{\tilde{u}}| - 1)^2 + k_{\tilde{v}}(|S_{\tilde{v}}| - 1)^2\}, \quad (3)$$

where

$$[\tilde{u} \ \tilde{v}] = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} [u \ v], \quad (4)$$

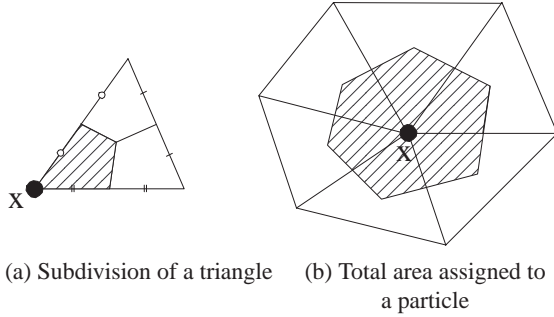
and  $k_{\tilde{u}}$  and  $k_{\tilde{v}}$  are the *stretch* stiffness constants along the  $\tilde{u}$

<sup>§</sup> The stretch model presented here differs from Baraff and Witkin's formulation in that it applies only to positive stretching. Negative stretching (i.e. compression) is handled by the bending interaction model described in Section 3.2.

<sup>¶</sup> We note that the shear stretch energy in Equation 3 can overlap with the pure stretch energy in Equation 2 in cases such as uniform extension. But the artifact from this overlapping is hardly observable since the stretch stiffness is very high in general and thus the cloth remains almost unstretched all the time.

and  $\bar{v}$  directions, respectively. The expressions for  $S_{\bar{u}}$  and  $S_{\bar{v}}$  are similar to those for  $S_u$  and  $S_v$  in Equation 1.

Anisotropy is an important characteristic of cloth, which exists in both stretching and bending deformations. Our unifying approach for stretch and shear intrinsically represents anisotropy in stretch deformations.



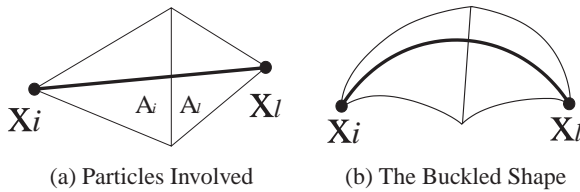
**Figure 1:** Finding the energy assigned to a particle

Equations 2 and 3 tell us the internal energy residing in a non-zero *area*; hence, the space-derivatives of these equations give the *areal* forces resisting the current stretch or shear deformation. However, given that we are working in a particle-based framework, we need to calculate the forces that act on the particles. This problem can be solved by subdividing each triangle into three equi-area subregions, as shown in Figure 1 (a), and condensing the energy of each subregion onto the corresponding particle. Therefore, the energy  $E_x$  associated with particle  $x$  is given by the sum of the areal energy of the shaded region in Figure 1 (b), and is expressed as

$$E_x = \frac{1}{3} \sum_{\tau \in T(x)} E_\tau \quad (5)$$

where  $T(x)$  is the set of triangles that share the particle  $x$ , and  $E_\tau$  is the areal energy of the triangle  $\tau$ . This conversion formula works for both stretch and shear energy functions.

### 3.2. Bending Interaction Model



**Figure 2:** Bending interaction model

The bending resistance is defined for every pair of triangles that share an edge, as shown in Figure 2(a). This interaction model is responsible for the post-buckling response

created by the compressive and bending forces. We adopt a bending interaction model that is basically the same as that proposed by Choi and Ko<sup>5</sup>, but with some differences described below. We first predict the buckled shape and then formulate the deformation energy function corresponding to the predicted shape.

Two adjacent triangles that are initially in the same plane are regarded as buckled if  $|x_l - x_i|$  in Figure 2(a) is shorter than the initial length  $L$ . We predict the buckled shape of the two triangles to be a singly curved surface (Figure 2(b)) with a constant curvature

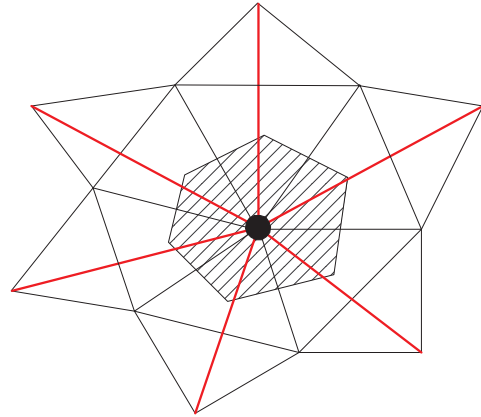
$$\kappa = \frac{2}{L} \text{sinc}^{-1} \left( \frac{|x_l - x_i|}{L} \right), \quad (6)$$

where  $\text{sinc}(x) = \frac{\sin x}{x}$ . Then, the bending deformation energy is defined as

$$E_B = \frac{1}{2} \int_{A_i + A_l} M \kappa da = \frac{1}{2} (A_i + A_l) M \kappa \quad (7)$$

where  $M$  is the bending moment, and  $A_i$  and  $A_l$  are the areas of the two triangles associated with  $x_i$  and  $x_l$ , respectively.

The feature distinguishing the above model from the original model is that the energy function contains the area term, which takes care of the irregularities in the triangular mesh.



**Figure 3:** Total bending energy assigned to a particle

As for the stretch and shear energy, we must convert the above area-associated energy to the particle-associated energy. Assuming that the energy is proportional to the area, we can divide the area-associated energy  $E_B$  into two parts, one for each triangle. We perform the procedure for every pair of triangles. Then, we can use an approach similar to Equation 5 to obtain the particle-associated bending energy by summing the bending energy contributions from all the participating triangle pairs, as shown in Figure 3.

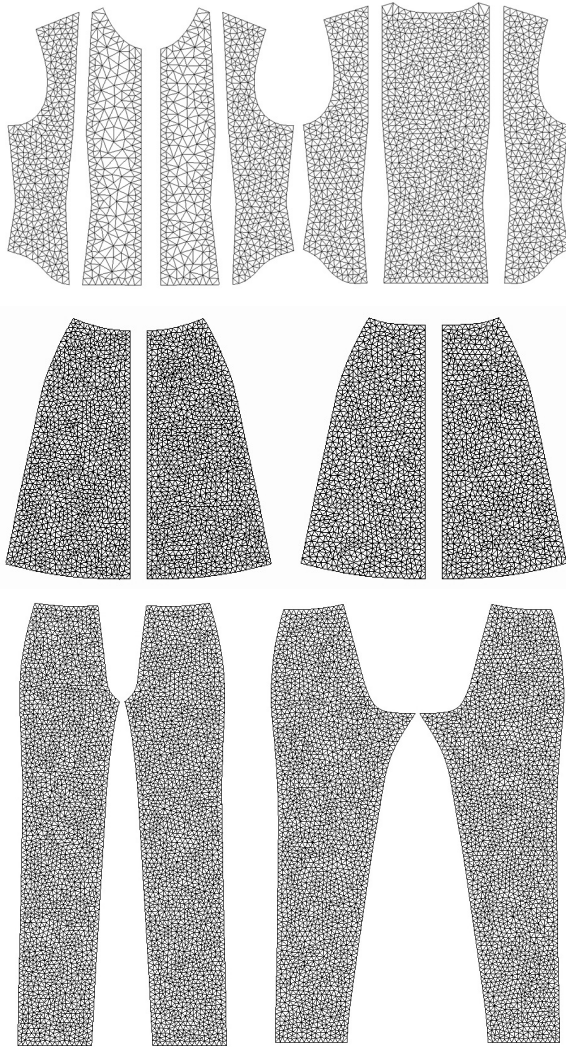
The bending anisotropy is handled by elliptically interpo-

lating the bending stiffness in the  $u$  and  $v$  directions.

$$k = \sqrt{\frac{k_u^2 u_{ij}^2 + k_v^2 v_{ij}^2}{u_{ij}^2 + v_{ij}^2}} \quad (8)$$

#### 4. Result

We implemented the techniques proposed in this paper on a Pentium 4 PC platform, and produced the animation sequences discussed below.



**Figure 4:** The internal representation of clothes; the patterns in the left correspond to the front part of the garments, and those in the right correspond to the back part.

Applying our proposed approaches for modeling buckling and anisotropy, we simulated two-pieces and pants designed by a professional fashion designer. Figure 4 shows

the internal representation of the clothes with irregular triangle meshes presented in Section 3. Though we used a constrained Delaunay triangulation method for the meshes used in the simulation, the proposed model is not restricted to a specific triangulation method. The images at the upper two rows in Figure 5 are snapshots taken from a walking animation in which the character is wearing two-pieces. The wrinkles generated by the simulation were natural due to the immediate buckling model and the artifacts caused by the irregularity of the meshes were not observed. The pants simulation result shown at the bottom row in Figure 5 also proved the proposed technique can handle the irregular triangular meshes quite well without noticeable artifacts.

#### 5. Conclusion

In this study, we extended Choi and Ko's immediate buckling model to irregular triangular meshes. The resulting model allows clothing to be constructed from arbitrary 2D shapes, which tremendously increases the applicability of the technique in areas such as fashion design. The extended model well handles the irregularities in the triangular meshes and produces natural and smooth wrinkles.

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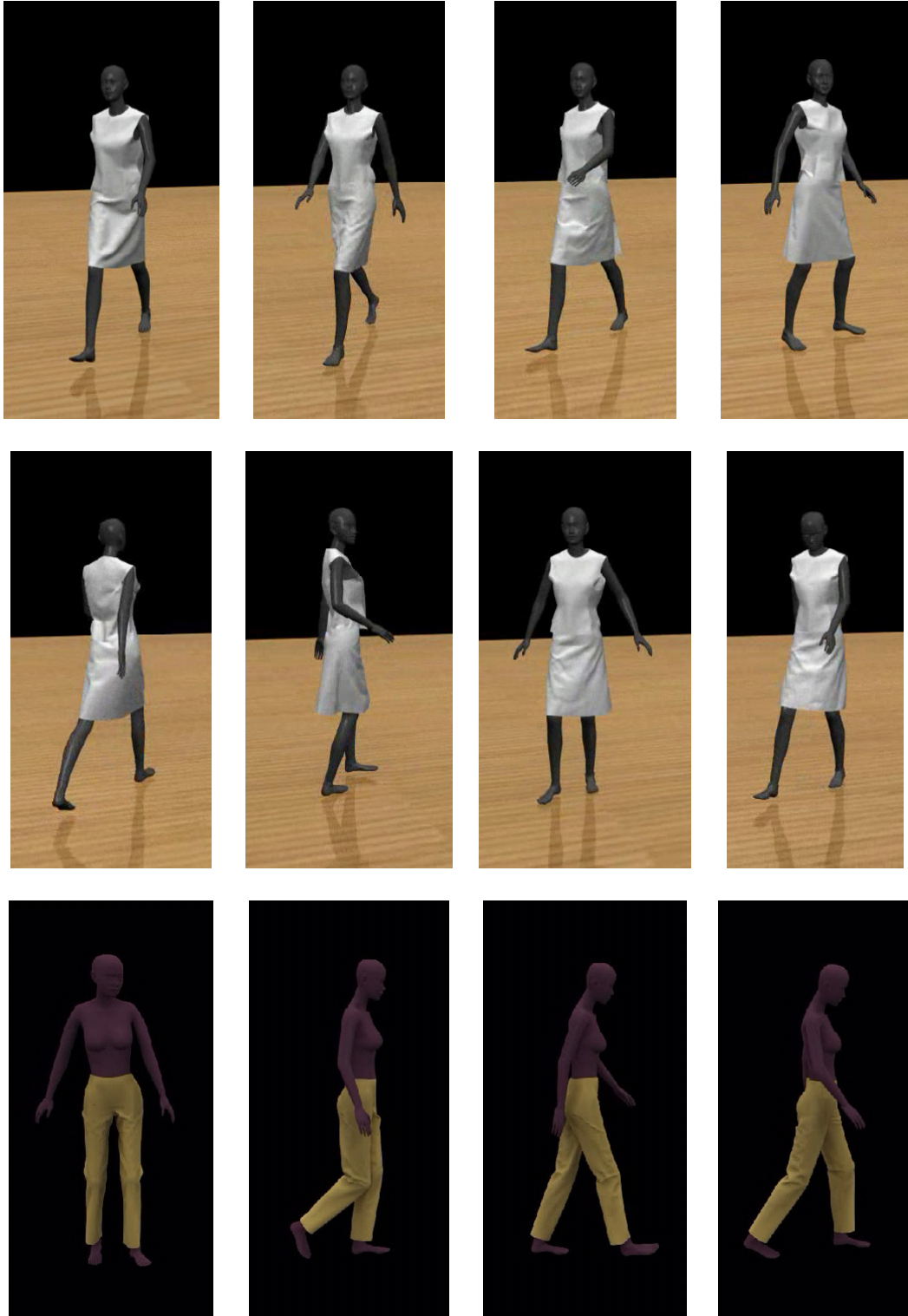


Figure 5: Simulation Results