Visibility Complexity of a Region in Flatland

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Abstract
The aim of this paper is to study the visibility complexity of different regions in a 2D scene. Based on mutual information, which we used in our previous work to define scene complexity, we propose two measures that quantify the complexity of a region from two different points of view. The knowledge of the complexity of a region can be useful to determine how difficult it is to recompute the visibility links for an animation depending on the regions visited or to obtain the complexity of the movement of a robot. We also envisage its applicability to obtain an optimal load balancing in a parallel computation by dividing the geometry in equal complexity regions.

Keywords: complexity, information theory, Monte Carlo, visibility

1. Introduction

Several complexity measures have been introduced from different areas to quantify the degree of structure or correlation of a system [1, 9, 7]. In a 3D scene, the complexity measure that we have proposed in our previous work [4, 5] is scene mutual information, which can be considered as the difficulty of computing accurately the visibility and radiosity in a scene. Scene mutual information, which is an information theory measure, quantifies the average information transport in a scene and the correlation among all their points or patches.

In our previous work we obtained a global complexity measure of a scene. In contrast, in this paper we apply this approach to study the visibility complexity of a region in flatland. The definitions introduced can easily be generalized to 3D scenes. In [6] we defined the complexity of animation and justified a higher complexity because of traversing a more complex "region". Here we will give two related measures to quantify the complexity of a region. Some potential applications of these measures are determining how difficult it is to recompute the visibility for an animation or obtaining the complexity of the movement of a robot. We think it could also be applied to obtain an optimal load balancing in a parallel computation by dividing the geometry in equal complexity regions.

The organisation of this paper is as follows: In section 2 we present the concept of scene visibility complexity applied to flatland. In section 3 we define two measures that quantify the complexity of a region in a scene. In section 4 we calculate the complexity of a region in several scenes and discuss the results obtained. Finally, in section 5, we present our conclusions and future work.

2. Framework

The most basic information theory definitions [3, 2] applied to 3D scene visibility were presented in [4]. In this section, entropy rate and mutual information are adapted to flatland by only changing the area of each patch with the length of each patch (see [11] for details). Flatland visibility and form factors are studied in [8, 10]. Thus, the *scene visibility entropy rate*, or simply *scene visibility entropy*, is defined by

$$H_s = -\sum_{i=1}^{n_p} \frac{L_i}{L_T} \sum_{j=1}^{n_p} F_{ij} \log F_{ij} \tag{1}$$

where n_p is the number of patches (2D segments), F_{ij} is the form factor between the patches i and j, L_i is the length of patch i and L_T is the total length of the scene (the sum of segment lengths). The entropy rate measures the average uncertainty that remains about the patch j visited next when an imaginary particle undergoing an infinite random walk, with the form factors as transition probabilities, is known to be on a given patch i.

The *discrete scene visibility mutual information* is defined by

$$I_s = \sum_{i=1}^{n_p} \sum_{j=1}^{n_p} \frac{L_i F_{ij}}{L_T} \log \frac{F_{ij} L_T}{L_j} \tag{2}$$

and can be interpreted as the average amount of information that the destination patch conveys about the source patch, and vice versa. Consequently, I_s *is a measure of the average information transfer in a scene*.

The *continuous mutual information* is given by

$$I_s^c = \int_{x\in\mathcal{L}} \int_{y\in\mathcal{L}} \frac{1}{L_T} F(x,y) \log(L_T F(x,y)) dxdy \tag{3}$$

where $\mathcal{L}$ is the set of segments that form the environment, x and y are points on segments of the environment and $F(x,y)$ is the differential form factor between x and y. This integral can be solved by Monte Carlo integration. Similarly to [4], the computation can be done efficiently by casting global lines uniformly distributed upon segments [12]. Thus, continuous mutual information can be approximated by

$$\begin{aligned} I_s^c &\simeq \frac{1}{N} \sum_{k=1}^{N} \log(L_T F(x_k, y_k)) \\ &= \frac{1}{N} \sum_{k=1}^{N} \log(\frac{L_T cos\theta_x cos\theta_y}{2d(x,y)}) \end{aligned} \tag{4}$$

where θ_x and θ_y are the angles which the normals at x and y form with the segment joining x and y, $d(x,y)$ is the distance between x and y, and N is the total number of

pairs of points considered, which is the total number of intersections

divided by two.

In [4, 5] continuous scene visibility mutual information has been proposed as an absolute measure of the complexity of scene visibility and discrete mutual information as a complexity measure of discretised scene visibility. We have also shown that when a patch is refined into m subpatches discrete mutual information increases or remains the same, and continuous mutual information of a scene is the least upper bound to discrete mutual information: $I_s \leq I_s^c$. We also established two proposals which show a close relationship between complexity and discretization: (i) the greater the complexity the more difficult it is to get a discretization which expresses with precision the visibility or radiosity of a scene and (ii) among different discretizations of a scene the best is the one with the highest discrete mutual information. Thus, while continuous mutual information expresses how difficult it is to discretise a scene to compute accurately the visibility, discrete mutual information gives us a measure of how well we have done it.

3. Visibility complexity of a region

Contrasting with the global complexity measure I_s^c introduced in our previous work, in this section we define two measures that consider the complexity of a part of a scene. On the one hand, we study the complexity of a set of segments contained in a region and, on the other, the complexity of a region contained between segments.

3.1. Complexity of a set of segments

As we have seen in the previous section, the continuous mutual information I_s^c can be computed approximately by casting global lines uniformly distributed. Each term

$$\log(\frac{L_T cos\theta_x cos\theta_y}{2d(x,y)}) \tag{5}$$

can be interpreted as the *information exchange between the points x and y*. Thus, I_s^c is obtained by the average of the information transfer of all the pairs of points connected by global lines and it represents the average information transport in a scene.

From this point of view, we can define the *continuous mutual information matrix* as formed by the terms

$$(I_s^c)_{s_i s_j} \simeq \frac{1}{N} \sum_{k=1}^{N_{s_i s_j}} \log(\frac{L_T cos\theta_x cos\theta_y}{2d(x,y)}) \tag{6}$$

where N is the total number of pairs of points considered, s_i and s_j represent two sets of segments of a scene and $N_{s_i s_j}$ is the number of lines which intersect at the same time the sets of segments s_i and s_j. A term $(I_s^c)_{s_i s_i}$ expresses the interaction between themselves of a set of segments and a term $(I_s^c)_{s_i s_j}$, with $i \neq j$, expresses the interaction between two different sets of segments. Note that this definition includes the particular case of single segment sets.

We also define the contribution of a set of segments s_i to the global complexity I_s^c as

$$(I_s^c)_{s_i} \simeq \frac{1}{N} \sum_{k=1}^{N_{s_i}} \log(\frac{L_T cos\theta_x cos\theta_y}{2d(x,y)}) \tag{7}$$

where N_{s_i} is the number of lines which intersect the set of segments s_i. Thus, $(I_s^c)_{s_i}$ is the sum of the elements of row i of the matrix. We consider $(I_s^c)_{s_i}$ as the *complexity of the set of segments s_i* and can be interpreted as the total information transferred by this set.

As we have mentioned, we think that this measure has a potential application in obtaining an optimal load balancing in a parallel computation.

3.2. Complexity of a region

In this approach, we consider the complexity of a region by computing the complexity of (ideally) all the points in this region (see below). In the experimental results presented in the next section, we take a squared grid, as small as we want, and we simply compute the complexity in the central point of each square (see figure 1). Thus, we obtain a complexity map of a region.

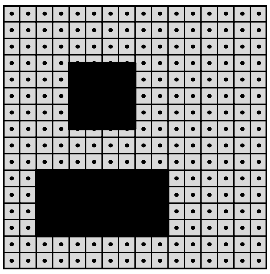


Figure 1: *A squared grid is used in order to compute the complexity in the central point of each square.*

Given a point x in a region, we can compute the complexity in this point by casting random lines from it in all directions. If we consider an infinitely small circle centered in x, we define that each line contributes to the complexity with a value

$$\log(\frac{L_T cos\theta_x cos\theta_y}{2d(x,y)}) = \log(\frac{L_T cos\theta_y}{2d(x,y)}) \quad (8)$$

where $cos\theta_x = 1$, because the line from the circle center is always normal to it (see figure 2).

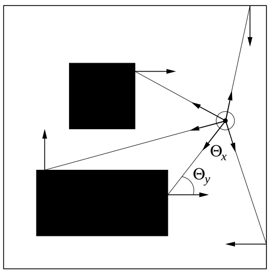


Figure 2: *Lines from a point. θ_x is always zero.*

With the average of the complexity values of all the lines cast, we get the *complexity in point x*

$$C_p \simeq \frac{1}{N}\sum_{k=1}^{N}\log(\frac{L_T cos\theta_y}{2d(x,y)}) \quad (9)$$

where N is the number of lines cast. It is important to remark that a singularity is produced when we take a point on a segment: we have only to consider points between segments. Another point of view could be taken from global lines traversing the grid squares.

As we will see in the next section, it can be interpreted that in a more complex region it will be more costly to insert an object than in a less complex one. And also, the more complex the region the more complex the animation in this region.

4. Results and discussion

Some preliminary results are presented in order to illustrate the behaviour of the measures introduced.

With respect to the complexity of a set of segments, we show in table 1 the values obtained for a scene with a rectangle and a square in its interior (figure 3). We can see that the set of segments in region 3 has a higher complexity with an important contribution of the interaction between the segments of this region and themselves. In contrast, in region 1, the complexity is the lowest and the total contribution of the interaction with the other sets of segments is more important than the interaction between themselves.

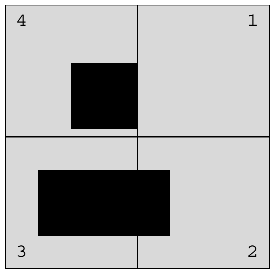


Figure 3: *A scene with a rectangle and a square in its interior. Four regions have been labeled.*

In order to analyze the complexity of a region, we compute this in six different scenes (figure 4). For each scene we show the colour map which illustrates the complexity of each region. The highest complexity corresponds to the red colour (or the darkest part in a black and white image) and the lowest complexity to the blue colour (or the lightest part in a black and white image). In these figures we specify the range of complexity obtained in each scene. As we can see, high complexity is found near the objects, walls, in the corners, and especially in the narrow spaces.

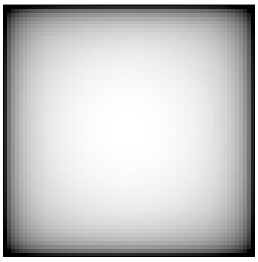
Another experiment is designed to test the increase in visibility complexity I_s^c when we insert an object in a region. In figure 5, a hexagon is situated in five different places corresponding to five different complexities of a region (see figure 4(c)). In table 2, we observe the perfect concordance between the two measures: the higher the complexity of a region the higher the increase in visibility complexity.

To compare the complexity of a region with the cost of the animation, in figure 6(b) we present a scene with two alternative animations. From the colour map of scene 6(a), we hope that the animation represented by a continuous line is more complex because it traverses a more complex region. If we measure the complexity of these two animations using the animation complexity measure C_a defined in [6], we observe the concordance between this measure and the complexity of a region: C_a values are 0.2413 for the continuous line and 0.1263 for the dashed line. In conclusion, traversing

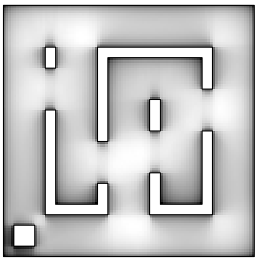
region	1	2	3	4	$(I_S^c)_{s_i}$	%
1	0.110040	0.057693	0.013321	0.089411	0.270465	10.74
2	0.057693	0.370869	0.036895	0.051278	0.516735	20.53
3	0.013321	0.036895	0.751718	0.161631	0.963565	38.28
4	0.089411	0.051279	0.161631	0.464096	0.766417	30.45
					2.517182	100.00

Table 1: *Contribution of each region in figure 3 to the global complexity I_S^c.*

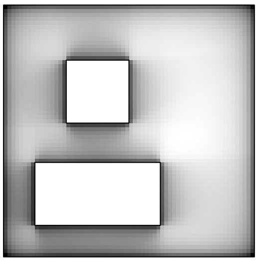
complex regions will yield a higher value in the animation complexity measure.



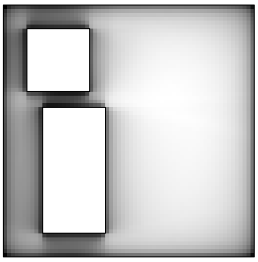
(a) [1.693, 5.100]



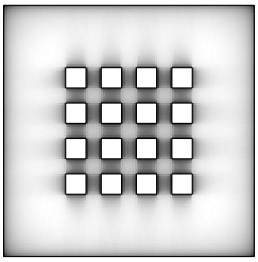
(b) [3.836, 7.312]



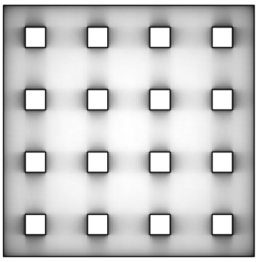
(c) [2.723, 5.981]



(d) [2.673, 6.064]



(e) [3.625, 7.330]



(f) [3.543, 7.531]

Figure 4: *Complexity colour map and range of the complexity of a region for six scenes. Different grids have been used with 10^3 lines cast from each grid cell center.*

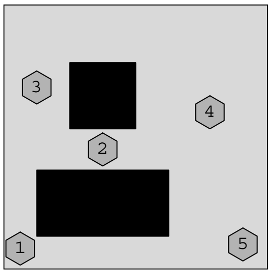


Figure 5: *A hexagon is situated in figure 4(c) in five different places.*

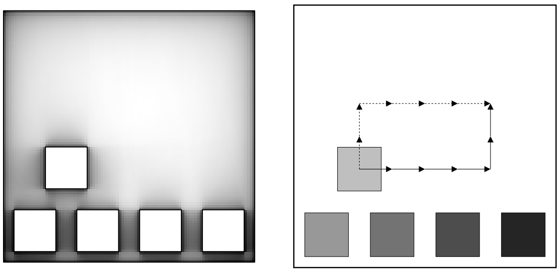
case	I_S^c	Δ	%
	2.517182	0	0
1	2.860125	0.342943	13.62
2	2.795483	0.278301	11.06
3	2.733251	0.216069	8.58
4	2.662806	0.145624	5.79
5	2.762630	0.245448	9.75

Table 2: *I_S^c values corresponding to figure 5 where an hexagon is situated in five different positions. The increase in scene visibility complexity is showed in each case (1-5) with respect to the reference scene 4(c), given in first row.*

5. Conclusions and future work

Based on mutual information we have defined the complexity of a region of a scene from two different but complementary points of view. We have presented preliminary results that show the possibilities of this approach and the relationship between the complexity of a region and the animation complexity in this region.

Future work will be addressed to incorporate radiosity into our complexity measure of a region, in the way indicated



(a) [2.697, 6.880] (b)

Figure 6: *(a) Complexity colour map and range of the complexity of a region. (b) Two alternative animations in figure (a) are represented by a continuous line and a dashed line.*

in [4], and to test the validity of our approach for optimal load balancing of raytracing a scene in a parallel computation.

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