# A Semi-empirical Model for the Representation of Materials in Photorealistic Rendering 

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#### Abstract

From the beginning of Computer Graphics various models intended to describe the optical properties of different materials have been proposed. Some of these models are purely empirical [1]. Others are very close to physics but they have a large computational overhead [2,3]. In this paper we introduce a new model for the representation of different materials in photorealistic rendering which is based on the microfacet approach and tailored to ray tracing. The proposed ideas are based on an equation developed by Hall and Greenberg [4]. The proposed model fills the gap between existing very primitive and highly sophisticated models, and because it is fast to compute and easy to understand it is not only interesting for scientists but also for practical purposes in industry. At the end of this paper several step by step improvements are described which will be integrated in the proposed equation in the near future.


We focus on the question of how the ideas of the microfacet approach [5-7] can be used in ray tracing. We suggest the following ray tracing equation (Figures 1 and 2 describe the underlying geometry):
$\mathrm{J}_{\mathrm{r}, \lambda}(\mathrm{P})=$
$\sum_{i=1}^{n}\left(k_{d f} F_{\lambda}\left(0^{\circ}\right) \cos \alpha_{i}\right.$
$\left.+k_{r g} F_{\lambda}\left(\varphi_{i}, n_{21}\right) e^{-c^{2} \delta_{i}^{2}}\right) J_{e, \lambda, i} f\left(r_{i}\right)$
$+\mathrm{k}_{\mathrm{df}} \mathrm{F}_{\lambda}\left(0^{\circ}\right) \mathrm{J}_{\mathrm{ab}, \lambda}$
$+k_{r g} F_{\lambda}\left(\theta, n_{21}\right) J_{\text {refl }, \lambda}\left(P_{\text {refl }}\right) f\left(r_{\text {refi }}\right) e^{-K_{\text {reff }, ~} r_{\text {refl }}}$
$+k_{\text {tp, rg }}\left(1-F_{\lambda}\left(\beta, n_{12}\right)\right) J_{\text {trans }, \lambda}\left(P_{\text {trans }}\right) f\left(r_{\text {trans }}\right) e^{-\mathrm{K}_{\text {trans }, \lambda} r_{\text {trans }}}$
$+\sum_{j=1}^{m}\left(k_{t p, d f} F_{\lambda}\left(0^{\circ}\right) \cos \alpha_{t p, j}\right.$

$$
\begin{equation*}
\left.+\mathrm{k}_{\mathrm{tp}, \mathrm{r} \mathrm{~g}}\left(1-\mathrm{F}_{\lambda}\left(\varphi_{\mathrm{tp}, \mathrm{j}}, \mathrm{n}_{12}\right)\right) \mathrm{e}^{-\mathrm{c}^{2} \delta_{\mathrm{t}, \mathrm{j}}^{2}}\right) \mathrm{J}_{\mathrm{e}, \lambda, \mathrm{j}} \mathrm{f}\left(\mathrm{r}_{\mathrm{j}}\right) \tag{C5}
\end{equation*}
$$

where:
$\mathrm{J}_{\mathrm{r}, \lambda}(\mathrm{P}) \quad:$ intensity value reflected from point $P$ to the viewer at wavelength $\lambda$
$\mathrm{J}_{\mathrm{ab}, \lambda} \quad:$ intensity value of the incoming ambient light

| $\mathrm{J}_{\mathrm{e}, \lambda, \mathrm{i}}$ | : intensity value of the light which is emitted from light source i |
| :---: | :---: |
| $\mathrm{J}_{\text {refl }, \lambda}\left(\mathrm{P}_{\text {refl }}\right)$ | : intensity value which is emitted from point $\mathrm{P}_{\text {refl }}$ (cf. Figure 2) |
| $\mathrm{J}_{\text {trans }, \lambda}\left(\mathrm{P}_{\text {rtrans }}\right)$ | : intensity value which is emitted from point $\mathrm{P}_{\text {trans }}$ (cf. Figure 2) |
| $\mathrm{k}_{\mathrm{df}}, \mathrm{k}_{\mathrm{rg}}$ | : diffuse and specular weighting coefficients regarding reflection |
| $\mathrm{k}_{\mathrm{tp}, \mathrm{df}}, \mathrm{k}_{\mathrm{tp}, \mathrm{rg}}$ | : diffuse and specular weighting coefficients regarding transmission |
| c | : roughness parameter |
| $\mathrm{n}_{21}\left(\mathrm{n}_{12}\right)$ | : relative refraction index: $\mathrm{n}_{21}=\mathrm{n}_{2} / \mathrm{n}_{1} \quad\left(\mathrm{n}_{12}=\mathrm{n}_{1} / \mathrm{n}_{2}\right)$ |
| $\mathrm{r}_{\mathrm{i}}$ | : distance between P and the light source i |
| $\mathrm{r}_{\text {refl }}\left(\mathrm{r}_{\text {trans }}\right)$ | : distance between P and $\mathrm{P}_{\text {refl }}(\mathrm{P}$ and $\mathrm{P}_{\text {trans }}$ ); cf. Figure 2 |
| $\mathrm{f}(\mathrm{r})$ | : distance function $\left(\mathrm{f}(\mathrm{r})=1 /\left(\mathrm{c}_{1} \mathrm{r}^{2}+\mathrm{c}_{2}\right)\right)$ |
| $\mathrm{K}_{\text {refl }, \lambda}\left(\mathrm{K}_{\text {trans }, \lambda}\right)$ | : extinction coefficient of medium 1 (medium 2) |
| $\mathrm{F}_{\lambda}\left(0^{\circ}\right)$ | : Fresnel equations for normal incidence |
| $\begin{aligned} & \mathrm{F}_{\lambda}\left(\varphi, \mathrm{n}_{21}\right) \\ & \exp \left(-\mathrm{c}^{2} \delta_{\mathrm{i}}^{2}\right) \end{aligned}$ | : Fresnel equations <br> : microfacet distribution function |
| $0 \leq \mathrm{k}_{\mathrm{df}}, \mathrm{k}_{\mathrm{rg}}, \mathrm{k}_{\mathrm{tp}, \mathrm{rg}}, \mathrm{k}_{\mathrm{tp}, \mathrm{df}} \leq 1$, |  |
| $\mathrm{k}_{\mathrm{df}}+\mathrm{k}_{\mathrm{rg}}=1$, | $\mathrm{k}_{\mathrm{tp}, \mathrm{df}}+\mathrm{k}_{\mathrm{tp}, \mathrm{rg}}=1$ |
| $\cos \alpha_{i}=\mathbf{n} \mathbf{l}_{i}$, | $\cos \varphi_{\mathrm{i}}=\mathbf{l}_{\mathrm{i}} \mathbf{h}_{\mathrm{i}}, \quad \cos \delta_{\mathrm{i}}=\mathbf{n} \mathbf{h}_{\mathrm{i}}$ |
| $\cos \alpha_{\mathrm{tp}, \mathrm{j}}=-\mathbf{n}$ | $\cos \varphi_{\mathrm{tp}, \mathrm{j}}=-\mathbf{h}_{\mathrm{tp}, \mathrm{j}} \mathbf{l}_{\mathrm{j}}, \quad \cos \delta_{\mathrm{tp}, \mathrm{j}}=\mathbf{n} \mathbf{h}_{\mathrm{t}, \mathrm{j}}$ |



Figure 1: The left picture shows the reflection of light at a facet (with halfway vector $\mathbf{h}=(\mathbf{l}+\mathbf{b}) /|\mathbf{l}+\mathbf{b}|)$, the right picture shows the refraction of light at the surface of a transparent facet; $\mathbf{h}_{\mathrm{tp}}=\left(-\mathbf{b}-\mathrm{n}_{21} \mathbf{l}\right) /\left(\mathrm{n}_{21}-\mathbf{l}\right)($ not normalized) [8]


Figure 2: Geometry of ray tracing: A ray which starts from the viewer ( $\mathbf{b}$ is the vector in the direction of the viewer) hits an object at point $\mathrm{P} . \mathrm{P}$ is illuminated by light sources (intensity values: $\mathrm{J}_{\mathrm{e}, \lambda_{,} \mathrm{i}}$ ), the intensity value $\mathrm{J}_{\text {refl }, \lambda}\left(\mathrm{P}_{\text {reff }}\right)$ which reaches P from the reflection direction $\mathbf{R}$ (this light is emitted from the point of intersection $\mathrm{P}_{\text {refl }}$ of the reflected ray with another object), and the intensity value $\mathrm{J}_{\text {trans }, \lambda}\left(\mathrm{P}_{\text {trans }}\right)$ which reaches P from the transmission direction $\mathbf{T}$.

The equation consists of six components C1 to C6:

- C1 describes the influence of the light sources (with angle of incidence $\alpha_{i} \leq 90^{\circ}$ ). As in the Cook/Torrance model [7] the Fresnel equations are used. This results in a realistic representation of the diffuse color and the highlight color. For materials with a small absorption coefficient a formulation of the Fresnel equations
should be used which was suggested by Blinn [9]. For materials with a large absorption coefficient (like metals) the following formulation of the Fresnel equations could be used (the following equation is a simplification of an existing equation [10,11]):
$\mathrm{F}_{\lambda}\left(\varphi, \mathrm{n}_{21}\right)=\frac{1}{2} \frac{(\mathrm{a}-\mathrm{Z} 0)^{2}+\mathrm{b}_{\mathrm{q}}}{(\mathrm{a}+\mathrm{Z} 0)^{2}+\mathrm{b}_{\mathrm{q}}}\left(1+\frac{(\mathrm{a}-\mathrm{Z} 3)^{2}+\mathrm{b}_{\mathrm{q}}}{(\mathrm{a}+\mathrm{Z} 3)^{2}+\mathrm{b}_{\mathrm{q}}}\right)$
where :
$\mathrm{Z} 0=\cos \varphi, \mathrm{Z} 1=\sin \varphi, \mathrm{Z1}_{\mathrm{q}}=\mathrm{Z} 1^{2}, \mathrm{Z} 2=\tan \varphi$,
$\mathrm{Z} 3=\mathrm{Z} 1 \mathrm{Z} 2, \mathrm{Z} 4=\mathrm{n}_{21}{ }^{2}, \mathrm{Z} 5=\kappa_{2}{ }^{2}, \mathrm{Z} 6=\mathrm{Z} 4(1-\mathrm{Z} 5)$,
$\left.\mathrm{Z} 7=\sqrt{(\mathrm{Z} 6-\mathrm{Z1}} \mathrm{q}_{\mathrm{q}}\right)^{2}+4(\mathrm{Z} 4)^{2} \mathrm{Z5}$,
$\mathrm{a}=\sqrt{\frac{\mathrm{Z} 7+\mathrm{Z6}-\mathrm{Z1} 1_{\mathrm{q}}}{2}}, \mathrm{~b}_{\mathrm{q}}=\frac{\mathrm{Z} 7-\mathrm{Z} 6+\mathrm{Z1} 1_{\mathrm{q}}}{2}$,
$\kappa_{2}$ : index of absorption of medium 2
- C 2 describes the influence of the ambient light out of medium 1. In this component the diffuse weighting coefficient $\mathrm{k}_{\mathrm{df}}$ is used because the influence of the ambient light will be much larger (resp. smaller) if the diffuse (resp. specular) reflection dominates, e. g. if $\mathrm{k}_{\mathrm{df}}$ is much larger (resp. smaller) than $\mathrm{k}_{\mathrm{rg}}$. Analogous to C 2 component C 6 describes the influence of the ambient light out of medium 2.
- C3 covers the influence of light which reaches P from the reflection direction. The extinction of light is represented by a wavelength dependent factor (Lambert's extinction law).
- C 4 covers the influence of light which reaches P from the transmission direction. It is very important to note that the Fresnel equations in this component have to be evaluated for the angle of refraction $\beta$ and the relative refraction index $n_{12}=n_{1} / n_{2}$ because we focus on light which travels from medium 2 into medium 1.
- C5 describes the influence of light sources (with angle of incidence $\alpha_{i}>90^{\circ}$ ). In this component we use the microfacet approach to describe transmission phenomena. This component is completely analogous to component C 1 .
- C6 describes the influence of the ambient light out of medium 2 analogous to C2.

Several improvements of the proposed equation are possible and much further work has to be done:

- More pictures have to be generated with the new equation and their quality and computation time have to be compared with images generated with the models developed by Phong [1], Cook/Torrance [7], and He [3] respectively.
- The presented ideas should be integrated into a combined radiosity and ray tracing method or distributed ray tracing should be used to yield good results for both smooth and rough surfaces.
- The computation time can be reduced if some very efficient approximations to the Fresnel equations are used which have been proposed by Pfeiffer [10,11]. These approximations are completely neglected in literature.
- The presented equation contains five parameters which cannot be computed from the optical properties of the modelled material: $\mathrm{k}_{\mathrm{df}}, \mathrm{k}_{\mathrm{rg}}, \mathrm{k}_{\mathrm{tp}, \mathrm{rg}}, \mathrm{k}_{\mathrm{tp}, \mathrm{df}}$ and c . They depend on the surface roughness $\sigma$, the wavelength $\lambda$ and the index of absorption $\kappa$ and could probably be at least approximately - computed from $\sigma, \lambda$ and $\kappa$ [12,13].
- The microfacet approach is only valid for rough surfaces (i. e. the ratio of the surface roughness $\sigma$ and the wavelength $\lambda$ must be greater than 1 ). But before replacing the microfacet approach by models based on wave optics it should be examined how the quality of the generated pictures is influenced or reduced due to the fact that the microfacet approach is theoretically not valid for smooth surfaces.
- A geometrical attenuation factor should be introduced $[3,14]$ and the laws of radiometry should be applied in order to model the off-specular peak phenomenon [5,6].


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