# SIG-based Curve Reconstruction

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**Abstract**

*We introduce a new method to compute the shape of an unstructured set of two-dimensional points. The algorithm exploits the to-date rarely used proximity-based graph called spheres-of-influence graph (SIG). We filter edges from the Delaunay triangulation belonging to the SIG as an initial graph and apply some additional processing plus elements from the Connect2D algorithm. This combination already shows improvements in curve reconstruction, yielding the best reconstruction accuracy compared to state-of-the-art algorithms from a recent comprehensive benchmark, and offers potential of further improvements.*

**CCS Concepts**

• ***Computing methodologies*** → ***Point-based models;***

## 1. Introduction

Reconstructing a curve based on given samples with no additional information other than the position is a difficult task, considering that no connectivity information is present. The reconstruction usually implies generating a graph on the input points and choosing specific edges to recreate the connectivity.

We introduce a new method for curve reconstruction, based on a combination of Delaunay triangulation (DT) and the *Spheres-of-influence graph* (SIG), which we present below. Our method is inspired by the Connect2D algorithm [OM13], replacing their starting graph, $BC_0$, with a SIG-based graph, and then enhancing it. That graph is then processed by inflating and sculpting, creating a final boundary for the input point set.

## 2. Method

The SIG has been introduced as a clustering method [Tou88]. In the SIG, two vertices are connected if the distance between them is less or equal to the sum of the distance to their respective nearest neighbours.

### 2.1. Initial graph computation

The algorithm first computes an intersection between SIG and Delaunay triangulation. By doing this, we combine the local proximity offered by SIG with the maximisation of minimum angle triangles provided by the Delaunay triangulation. This graph acts as the $BC_0$ initial graph in the Connect2D algorithm, which will be further processed by inflating and sculpting, which we explain next.

Visually, the SIG can be interpreted as centring a circle at each vertex whose radius is equal to the distance to its nearest neighbour, i.e., just touching this closest vertex. We connect all vertices whose circles intersect. This relation encodes spatial proximity without being limited by a fixed number of neighbours, such as $k-neighbourhood$.

Applying the next step, inflating, requires no isolated vertices and a single connected component, with a boundary that encloses the shape. SIG constrained to the DT does not however guaranteed to be connected or to enclose all points, and does not have a guarantee on vertex degree. In order to remedy this issue for inflating, we need to apply post-processing steps.

1. *Eliminate leaf nodes:* We increase the degree of the leaf nodes by adding their shortest incident Delaunay edge to the graph.
2. *Aim to make vertices manifold:* We decrease the degree of vertices to two where this does not create additional leaf vertices. Decreasing the degree is done by removing the longest incident edges that do not disconnect the graph until either the degree is two or no more edges can be removed.
3. *Connect to a single graph:* We use a disjoint set to connect the current graph to a single connected component. Delaunay edges that are not part of the graph are sorted in ascending order of the increase in the total boundary length. Hence, we add edges that connect the graph while keeping the total boundary length minimal.
4. *Close the boundary:* In rare cases of non-uniformly spaced samples, the SIG can leave holes in the boundary, resulting in a hollowed-out shape. We fix this as follows: we perform a breadth-first search starting from a vertex on the convex hull, marking all the encountered triangles in the DT as *outside*. The search will not visit any of the edges already part of the reconstruction. However, in the case of an open boundary, most of the triangles will be marked as *outside*. To mitigate against open





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boundaries, we employ a heuristic: we use a counter to keep track of the total number of *outside* triangles. While $> 70\%$ of triangles in the DT remain marked *outside*, we add edges incident to edges between *outside* triangles to the graph, performing the triangle search and *outside* marking again, and recomputing the counter. This has worked well in our experiments.

### 2.2. Inflating

Inflating makes non-conforming vertices (i.e. with degree more than two) manifold on the boundary by adding incident *outside* triangles. Candidates are sorted in a priority queue in ascending order by the increase in total boundary length which is computed by adding the length of new edges and subtracting the length of edges to be removed. We repeat this procedure until no more non-conforming vertices remain. However, by adding the edges of new triangles to the graph, some of the edges can become interior to the boundary. Hence, we remove any edge that is not incident to a triangle marked as *outside*.

### 2.3. Sculpting

The graph created by inflating will contain all vertices either on or interior to the boundary. These isolated interior vertices have to be exposed to the boundary so that the reconstructed boundary can interpolate all the points. Incident triangles to interior vertices are sorted by the same boundary length increase criterion as for inflating. When a candidate triangle is added to the graph, similarly to inflating, we **XOR** this triangle's edges with the current graph.

## 3. Results

We tested the proposed method against state-of-the-art curve reconstruction algorithms using the 2D Points Curve Reconstruction Survey and Benchmark [OPP*21]. We analysed results for the reconstruction of manifold curves (using the subset of the ground truth datasets that interpolate all points - approximately 1200 sets) and curves with sharp corners, both for edge-identical reconstruction compared to ground truth. On manifold data, our algorithm shows the best accuracy (90.2% compared to second best - 88.3%), while on sharp corners, it is the second best, achieving the same percentage as the original Connect2D, only superseded by GathanG [DW02] which is specialised for sharp corners' reconstruction.

Moreover, already the initial graph (SIG subset of the Delaunay triangulation with selected post-processing steps) produces competitive results, and it has the additional advantage of being able to reconstruct open curves and multiples curves as well. Detailed results are present in Figures 1 and 2.

## 4. Conclusion and Future Work

We propose a reconstruction method based on the SIG for boundary reconstruction of a set of two-dimensional points. The SIG improves on capturing proximity connections between the vertices, and does so without requiring a specific number of neighbours as a parameter. Together with steps from an existing method it correctly reconstructs the boundary of the input set in more cases than the state-of-the-art. Current results are promising, encouraging further improvements of this approach. Hence, our future work includes:

- Improving the boundary closing heuristic directly on the SIG where it leaves holes, to eliminate the current heuristic;
- Proving a sampling condition under which the algorithm guarantees reconstruction;
- Increasing the method's robustness for multiple curves, open curves and non-manifold inputs;
- Extending the algorithm to surface reconstruction.

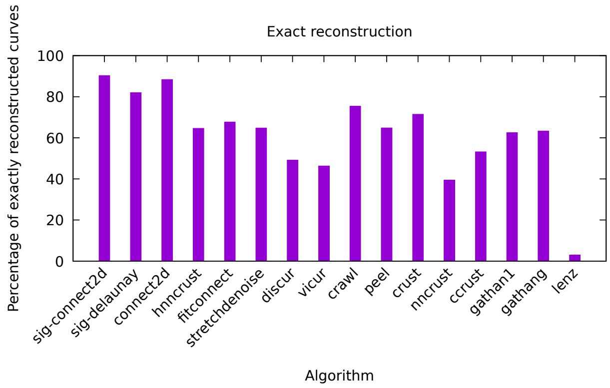


**Figure 1:** *Manifold reconstruction*

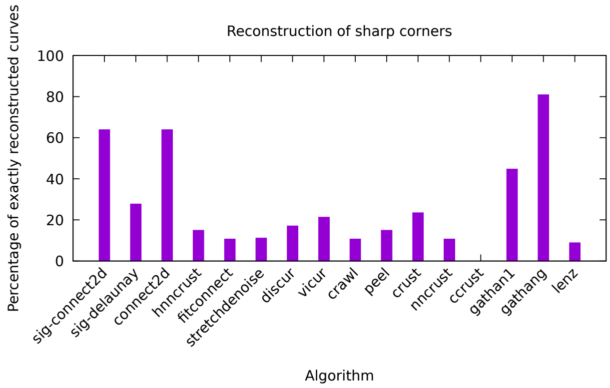


**Figure 2:** *Sharp corner reconstruction*

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