Conversion of CAD Models to Loop Subdivision Surfaces

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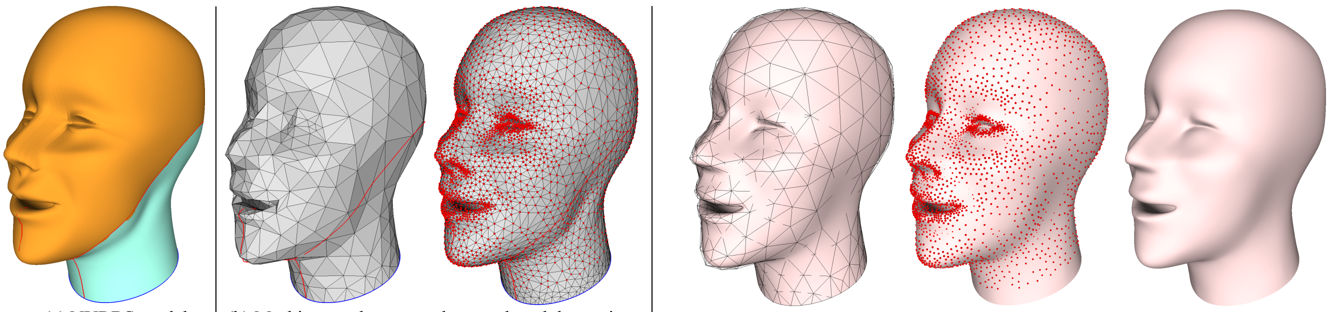

(a) NURBS model (b) Meshing results as template mesh and data points (c) Subdivision surface, with control mesh and with data points

Figure 1: *(a) Input model with two patches. (b) Data preparation via meshing the model. (c) Subdivision surface result.*

Abstract
We propose a general framework for converting a CAD model, which is a collection of trimmed NURBS surfaces, to a single Loop subdivision surface. We first apply a Delaunay-based meshing method to generate a template mesh and a set of data points from the input model, and then fit a Loop subdivision surface using exact evaluation.

1. Introduction

Tensor-product NURBS are widely used to represent shapes in Computer-Aided Design. Because of their restriction to the regular grid structure, a CAD model typically consists of a collection of trimmed NURBS surfaces stitched together with certain boundary management. In contrast, subdivision surfaces can model smooth surfaces of arbitrary topology and thus offer a promising alternative. We propose to convert a CAD model to a single watertight Loop subdivision surface [Loo87].

There is a tremendous amount of work about adaptive and feature-preserving triangular meshing of CAD models. Notable representatives include the advancing-front based surface mesh generation [TOC98], the particle-based approach [BLW12], and the methods based on Delaunay triangulation and refinement in 2D [CB97] and 3D [BDL09].

Also, plenty of work has been done on fitting a Loop subdivision surface to a given data set, e.g., a dense triangular mesh or a point cloud obtained from a shape. Hoppe et al. [HDD*] extended the original Loop subdivision scheme [Loo87] to include sharp features and introduced a smooth surface reconstruction method that uses a piecewise linear approximation of the subdivision surface for fitting error evaluation. Later, Marinov et al. [MK05] introduced an optimization method that uses the true representation of the limit surface. Ling et al. [LWY08] presented an exact evaluation of the limit surface with all types of sharp features. All these methods require (a) mesh simplification for a coarse control mesh (*the template mesh*), and (b) mesh topology optimization such as local refinement and edge flips.

We propose a conversion framework that starts directly from the CAD model, and combines the meshing and fitting processes. Using the meshing process of [BDL09] on the CAD model, we produce a coarse mesh that can be used as the template mesh without further mesh simplification or optimization. Additionally, the data points used for fitting are well distributed over the shape.

2. Conversion framework

There are two stages: data preparation and surface fitting.

Data preparation. Given a CAD model, we use the Delaunay refinement based method [BDL09] to generate triangular meshes. The resulting mesh is controlled by three criteria: the size and angle of the triangles, and the approximation error, ε, of the triangles to the shape. In our experiments, we vary ε to generate meshes from coarse to fine.

Using two thresholds, ε_1 and a smaller ε_2, we generate two meshes accordingly: (I) a coarse feature-preserving triangle mesh $\mathcal{M}_1$ as the template mesh for later fitting, (II) a relatively dense mesh $\mathcal{M}_2$ whose vertices are the data points X to fit, see Figure 1b. In our experiments, $\varepsilon_2 = 0.2\varepsilon_1$.

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Note that the Delaunay refinement process inserts new points into the triangulation, one at a time, to remove the triangles that do not meet some criteria. This means that we only introduce new points in places where they are needed most. In this way, the template mesh and the data points used for fitting improve the approximation error adaptively.

Also, as feature curves are contained in CAD models and can be easily detected, feature tags can be passed to the mesh vertices while meshing. Thus, there is no need to do further feature detection.

Surface fitting. In this stage, a subdivision surface is computed with the template mesh $\mathcal{M}_1$ and data points X from the data preparation stage. We use the exact evaluation of Loop subdivision surface [Sta] and adopt the optimization method proposed in [MK05].

Each triangle face f_i in $\mathcal{M}_1$ can be viewed as a smooth triangular patch defined by its neighboring control points. These patches compose the limit surface $\mathcal{S}$. For each data point $\mathbf{x}_i \in X$, we denote the parameter (referred to as *surface coordinate*) of its corresponding point on the limit surface $\tau_i = \langle i, (v_i, w_i)\rangle$, where (v_i, w_i) is the coordinate in the triangle domain of that patch (as defined in [Sta]).

The energy function has two parts: the data fitting error and the smoothness of the control mesh:

$$E(\mathbf{C}, \Upsilon) = \frac{1}{n}\sum_{i=0}^{n} \|\mathcal{S}(\tau_i) - \mathbf{x}_i\|^2 + \frac{\lambda}{m}\sum_{k=0}^{m} \left\|V(c_k)^T V(c_k)\right\|^2, \quad (1)$$

where $\Upsilon = \{\tau_i\}_{i=0,\dots,n}$ collects the data point coordinates, $\mathbf{C} = \{\mathbf{c}_k\}_{k=0,\dots,m}$ collects the control points in $\mathcal{M}_1$, $V(\cdot)$ is the discrete Laplacian, and $\lambda = 0.1$.

We optimize both the control points and surface coordinates [MK05]. For each data point, we use the tangent plane at its current surface coordinate to search for the closest point, and then perform coordinate correction. The initial coordinates are assigned by searching for the closest point on a discrete piecewise linear approximation of the surface.

3. Results and discussion

Our framework converts a CAD model composed of NURBS patches to a single watertight Loop subdivision surface.

We have tested our method on some simple models, see Figures 1 and 2. The input CAD models are shown on the left with each NURBS patch colored differently. The output subdivision surfaces are rendered using dense tessellations of their limit surfaces. Table 1 lists several statistics for the presented conversion results.

Limitations. As parameterization distortion exists around extraordinary vertices (EVs), i.e., vertices whose valency is not six, further work will investigate how to reduce the number of EVs in the resulting subdivision control meshes. Also, current surface evaluation around features is not accurate. This is why the sandal sole and the mechanical part have higher fitting error. The exact evaluation method of [LWY08] is expected to lead to smaller errors near features.

Model	#patches	#V	#F	n	Avg. error
Head	2	474	908	4551	0.00287
Sandal sole	3	357	710	2571	0.00815
Last	2	373	715	776	0.00273
Bulb	10	199	394	3027	0.00172
Mechanical part	14	244	484	2210	0.00586

Table 1: *The number of NURBS patches in the model, the number of vertices and faces of the template mesh $\mathcal{M}_1$, the number of data points, and the average fitting error.*

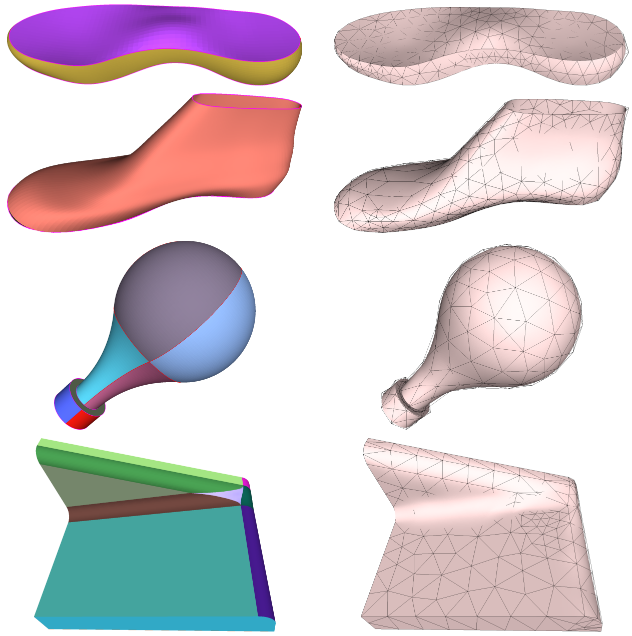

Figure 2: *More examples: sandal sole, last, bulb, and a mechanical part. Left: input NURBS patches. Right: fitted subdivision surfaces (the limit surfaces) and their control meshes.*

References

[BDL09] BUSARYEV O., DEY T. K., LEVINE J. A.: Repairing and meshing imperfect shapes with Delaunay refinement. In *SIAM/ACM Joint Conference on Geom. and Phys. Modeling* (2009), pp. 25–33. 1

[BLW12] BRONSON J. R., LEVINE J. A., WHITAKER R. T.: Particle systems for adaptive, isotropic meshing of CAD models. *Engineering with Computers 28*, 4 (2012), 331–344. 1

[CB97] CHEN H., BISHOP J.: Delaunay triangulation for curved surfaces. In *Proc. 6th Int. Meshing Roundtable* (1997), pp. 115–127. 1

[HDD*] HOPPE H., DEROSE T., DUCHAMP T., HALSTEAD M., JIN H., MCDONALD J., SCHWEITZER J., STUETZLE W.: Piecewise smooth surface reconstruction. In *ACM SIGGRAPH '94*, pp. 295–302. 1

[Loo87] LOOP C. T.: *Smooth Subdivision Surfaces Based on Triangles*. Master's thesis, Dept. of Mathematics, University of Utah, 1987. 1

[LWY08] LING R., WANG W., YAN D.: Fitting sharp features with Loop subdivision surfaces. In *Proc. of the Symposium on Geometry Processing* (2008), pp. 1383–1391. 1, 2

[MK05] MARINOV M., KOBBELT L.: Optimization methods for scattered data approximation with subdivision surfaces. *Graphical Models 67*, 5 (Sept. 2005), 452–473. 1, 2

[Sta] STAM J.: Evaluation of Loop subdivision surfaces. In *ACM SIGGRAPH '98 Course Notes*. 2

[TOC98] TRISTANO J. R., OWEN S. J., CANANN S. A.: Advancing front surface mesh generation in parametric space using a Riemannian surface definition. In *Proc. 7th Int. Meshing Roundtable* (1998), pp. 429–445. 1