

# Towards Image Rendering using models of Image Manifolds

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## Abstract

*Building computer representations of real objects, which could be archaeological artifacts or planned buildings, is a very active research area. Typically objects are represented in computer memory as a collection of geometric primitives. In this paper we build on the concept of Image Manifolds, which we model in image space. This enables us to model the appearance of an object essentially as a collection of images. We discuss one possible approach to modelling image manifolds, which uses partial differential equations.*

Categories and Subject Descriptors (according to ACM CCS): I.4.10 [Computer Graphics]: Image Representation

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## 1. Introduction

The concept of an *image manifold* is one that has become increasingly common in vision [BP94, LFHN98, DG05], and recently to some extent in the graphics literature. Images, which can be considered points in an high-dimensional *image space*, of an object or environment undergoing a series of transformations (which are typically parameterised in some way) are not randomly placed in image space. Instead under certain circumstances these can be approximated as an  $n$ -manifold, where  $n$  is the number of variables controlling the image transformations, which we call an *image manifold*. This mapping between the high-dimensional image space and a lower dimensional sub-space has been particularly useful in the area of vision [TP91, NNM96]. However developing efficient in-memory representations of image manifolds has proven hard, and many researchers have used techniques like PCA to simplify the representations. This is acceptable if the intended use is recognition of images. However for the purpose of synthesising images it proves to lose too many of the details that make images appear realistic to human observers. A potential solution to this problem is to develop existing surface representation techniques to model these structures in image space directly, thereby exactly representing all of the images that make up the manifold.

In this paper we present our work using PDE surfaces to model a selection of image manifolds. In Section 2 we outline our approach, and discuss our solutions to the resulting PDEs. In Section 3 we illustrate several example images from some of the image manifolds we are studying, and dis-

cuss some of their properties. We then test our model of the image manifold by evaluating some previously unseen points on this manifold, and visually inspecting the resulting images. In Section 4 we conclude with a discussion of the these images, and future directions of study these have opened.

## Contributions

Our work forms part of a growing body of literature which looks at the relationship between specific parameterised sets of images and the corresponding structures these form in image space [DG05, LFHN98, Ten98, WDCB05, HHLM07, Ver06]. We see our work as making the following contributions:

- We extend existing surface modelling techniques, which are typically oriented towards CAD style applications, to model image manifolds. This provides a theoretical foundation for producing visually plausible intermediate images on image manifolds, as opposed to existing heuristic methods such as morphing.
- Using several example image manifolds we show anecdotally, using visual evidence, that our models of image manifolds produce reasonable representations of image manifolds.

## 2. Approach

Typically the computer graphics problem of modelling a 2-D surface embedded in a 3-D space has been solved using

one of several alternative methods. Of these methods, the three that we are interested in are: polygon meshes, usually triangles; NURBS [PT97]; and within the last few years, the PDE surface method [UBW99].

For our work, we require a more general ‘manifold’ modelling technique, capable of modelling a structure parametrised by  $p$  parameters in a  $n$  dimensional space. To this end we have extended the standard definitions for PDE surfaces, which is briefly outlined in the remainder of this section.

### 2.1. PDE Surface

Initially here we will consider a 2-D parametric surface  $\underline{X}(u, v)$  in a 3-D space, which can be expressed as

$$\underline{X}(u, v) = (x(u, v), y(u, v), z(u, v)) \quad x, y, z \in \mathbb{R} \quad (1)$$

We now assume our surface to be periodic in  $v$ , and restrict it to the finite domain  $\Omega = \{u, v : 0 \leq u \leq 1; 0 \leq v \leq 2\pi\}$ . We choose, as is usually the case [UBW99, DQ07], to use an elliptic PDE. We therefore have that

$$\left( \frac{\partial^2}{\partial u^2} + \alpha^2 \frac{\partial^2}{\partial v^2} \right)^2 \underline{X}(u, v) = 0, \quad (2)$$

where  $\alpha$  is a “smoothing parameter”, which controls the length over which the boundary conditions influence the interior of the surface.

Existing work using the PDE surface method [UBW99] has used the method of separation of variables [CB78] to find an analytic solution to the PDE above, for which a computationally efficient implementation is also possible.

Given that Eqn. (2) is elliptic, the boundary conditions we have imposed are continuous, and the surface is “closed”, we use the following general solution:

$$\underline{X}(u, v) = \underline{A}_0(u) + \sum_{n=1}^{\infty} [\underline{A}_n(u) \cos(nv) + \underline{B}_n(u) \sin(nv)], \quad (3)$$

where  $\underline{A}_0$ ,  $\underline{A}_n$  and  $\underline{B}_n$  are vector valued functions as follows:

$$\underline{A}_0 = \underline{a}_{00} + \underline{a}_{01}u + \underline{a}_{02}u^2 + \underline{a}_{03}u^3, \quad (4)$$

$$\underline{A}_n = \underline{a}_{n1}e^{\alpha nu} + \underline{a}_{n2}ue^{\alpha nu} + \underline{a}_{n3}e^{-\alpha nu} + \underline{a}_{n4}ue^{-\alpha nu}, \quad (5)$$

$$\underline{B}_n = \underline{b}_{n1}e^{\alpha nu} + \underline{b}_{n2}ue^{\alpha nu} + \underline{b}_{n3}e^{-\alpha nu} + \underline{b}_{n4}ue^{-\alpha nu}. \quad (6)$$

Values for the vector constants  $\underline{a}_{00}, \dots, \underline{a}_{03}, \underline{a}_{n1}, \dots, \underline{a}_{n4}, \underline{b}_{n1}, \dots, \underline{b}_{n4}$  are determined by Fourier analysis of the boundary conditions imposed at  $u = 0$  and  $u = 1$ . In the examples presented here we have opted to truncate the Fourier series at  $N = 6$ , and introduce a remainder term,  $\mathbf{R}(u, v)$ . This approach is commonly seen in the literature [UBW99, KUW04] for cases where the boundary cannot be expressed precisely without an infinite series. For brevity, images with  $N = 8, N = 10$ , etc., are omitted here.

The image manifolds we study are not however 2-D surfaces embedded in a 3-D space. Typically images intended for viewing by humans have upwards of 300,000 pixels, each made up of 3 colour channels. In this case the surfaces we are dealing with are now embedded in a much higher dimensional space. We can extend the solution we outlined to deal with this scenario – one simply has to provide suitable boundary conditions for each dimension.

Many image manifolds are not parametrised by just two parameters and so we are required to extend our definition of the PDE surface to higher dimensional surfaces:

$$\left( \frac{\partial^2}{\partial u^2} + \alpha^2 \frac{\partial^2}{\partial v^2} + \beta^2 \frac{\partial^2}{\partial w^2} \right)^2 \underline{X}(u, v, w) = 0. \quad (7)$$

In cases with further additional parameters we can extend Eqn. (7) further still, although in the interest of brevity this is not discussed here.

However, the analytic solution previously outlined for Eqn. (2) cannot be used for Eqn. (7). We have therefore developed a suitable numeric scheme, discussed further in [WUL07].

### 3. Initial results



(a) Knight

(b) Teapot

**Figure 1:** Sample images from the image manifolds studied

Fig. 1 illustrates two examples of source images from the image manifolds we have been studying. These provide us with examples of features likely to be found in many different kinds of images. The features that we are particularly interested in are the source of the images (real or ray-traced), the transformations range from rotations and translations of a single object to illumination of a complex object. In the case of the Knight dataset each of the boundary conditions was formed from 9 images. For the teapot our boundary conditions were specified by 20 images.

In the Knight set the images vary in illumination, being illuminated from a range of points that form a hemisphere around the subject. We use  $\theta$  and  $\phi$  of the hemisphere as the  $u$  and  $v$  parameters for our image manifold, with  $u$  in  $[0, 1]$ , and  $v$  in  $[0, 2\pi]$ . In the teapot dataset  $u$  is a vertical translation of the image in  $[0, 1]$ , whilst  $v$  in  $[0, 2\pi]$  is a rotation around the vertical axis.

Some images we have generated from our models of the image manifolds are shown in Fig. 2. In almost all cases the images that we present are quite some distance in parameter space from the original sample images we used to build our model of the image manifold. This leads to some images containing large, obvious visual inconsistencies when compared with the original images. In these cases this results from our sample images being too far apart in image space. This in turn results in our models of the image manifold not accurately representing the manifold's actual path through image space. In the case of the Knight however we find that the path our model has taken in image space produces much more visually plausible results, and indeed our results here certainly seem comparable to those produced in the original Lightstage work from which the Knight images are taken.

This difference in success of the results can be explained through estimations of the curvature of the two corresponding image manifolds. In the case of the Knight we believe to have a much lower curvature in image space, and hence have sufficient samples that our models of the surface correspond closely with that of the true path in image space.

#### 4. Conclusion

The early results we have presented here are not sufficient to draw major conclusions from. The results here still show great promise, and as such further study is now proposed, looking at various as yet unconsidered factors. We intend to consider:

- Required number of Fourier modes, values of the parameter  $\alpha$  and the order of the PDE surfaces.
- The distance between sample images in both parameter space and image space, in particular through the estimation of the manifold's curvature.
- Better estimates of the derivatives of our surfaces.
- Other surface modelling techniques.

Currently all of our techniques assume that the underlying surface must pass through the points we have sampled. This is not always the case, because of for example the noise introduced by the CCD. This implies that a noisy image in image space is not simply a point as we have previously been assuming, but a point within high dimensional probability density field, centred around an unknown noise-free image. In order to work around this it is proposed that we move from a surface fitting approach which interpolates between images on the manifold, to a system whereby we opt to fit an approximate surface that is constrained in some fashion to pass near our sample images.

Throughout this work we have made one major assumption: image manifolds are continuous and differentiable. Up to this point we have not explicitly considered the fact that this will not always be the case. To convince yourself of this, consider a simple example of a previously occluded object passing into view. The first image where this new object appears will now be quite some distance in image space from

the last image in which it was occluded. A more subtle example where this may be a problem is that of rotation of a part of an object (e.g. the spout or handle of a teapot) which is mostly unlike the rest of the object. In this case a very small change in parameter space may result in a large movement in image space. Nonetheless there are still many interesting questions to be answered – “In these degenerate cases can a reasonable approximation be made by assuming they are both continuous and differentiable?”. Furthermore a general study of the topological properties of complex real image manifolds is a vast and interesting area for potential work.

Specular highlights and occlusions are just some of the many things found in complex real-world scenes that we have illustrated here. In our on-going work we have several other simpler synthetic datasets to illustrate important general features of images, as well as several other real-world sets.

We have so far chosen to only consider RGB colour images in this work, which is neither perceptually linear, nor immune to problems with variations in illumination. On the other hand the CIE  $L^*a^*b^*$  [Fai98] colour space is probably a more suitable colour space to use for this problem due to its perceptually linear nature, and larger gamut.

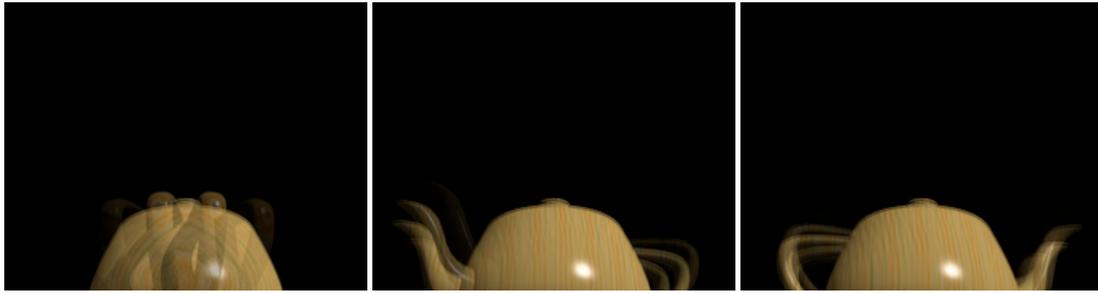
To date our work has not focused on optimisation or real-time rendering, and although it would probably be possible to accelerate rendering of lower resolution images from the PDE method using the analytic solution on graphics hardware this has not been addressed. It should also be noted that the analytic solution is only appropriate to the 2-D manifolds, and many of our data sets are of a much higher dimension than this.

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(a) Teapot (PDE:  $\alpha = 1, u = 0.5, v = 1.63$ ) (b) Teapot (PDE:  $\alpha = 1, u = 0.5, v = 3.26$ ) (c) Teapot (PDE:  $\alpha = 1, u = 0.5, v = 5.96$ )



(d) Knight (PDE:  $\alpha = 1, u = 0.5, v = 1.63$ ) (e) Knight (PDE:  $\alpha = 1, u = 0.5, v = 3.26$ ) (f) Knight (PDE:  $\alpha = 1, u = 0.5, v = 5.96$ )

**Figure 2:** Sample images from the image manifolds studied

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