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Multi-resolution Morphological Representation of Terrains

E. Danovaro, L. De Floriani, M.Vitali, L.Papaleo†

Dipartimento di Informatica e Scienze dell'Informazione, Universita' di Genova, Italy

Abstract

Mesh-based terrain representations provide accurate descriptions of a terrain, but fail in capturing its morphological structure. The morphology of a terrain is defined by its critical points and by the critical lines joining them, which form a so-called surface network. Besides being compact, a morphological terrain description supports a knowledge-based approach to the analysis, visualization and understanding of a terrain dataset. Moreover, because of the large size of current terrain data sets, a multi-resolution representation of the terrain morphology is crucial. Here, we address the problem of representing the morphology of a terrain at different resolutions. The basis of the multi-resolution terrain model, that we call a Multi-resolution Surface Network (MSN)*, is a generalization operator on a surface network, which produces a simplified representation incrementally. An MSN is combined with a multi-resolution mesh-based terrain model, which encompasses the terrain morphology at different resolutions. We show how variable-resolution representations can be extracted from an MSN, and we present also an implementation of an MSN in a compact encoding data structure.*

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Algorithms]: Terrain models, morphology, generalization, hierarchical models

1. Introduction

Terrain models, built from very large data sets provided by acquisition devices (e.g., satellite or aerial photos), are often excessively complex for applications such as real-time analysis and visualization. Thus, techniques for controlling the level of detail become crucial.

A terrain model consists of a finite set of points in a domain in the *x*-*y* plane at each of which an elevation value f is given. If the data points are regularly spaced in the domain, the terrain model is called a *Regular Square Grid* (RSG). Otherwise, the data points in the *x*-*y* plane are connected in a *triangle mesh*, which provides a piecewise-linear interpolating function to the terrain data, called a *Triangular Irregular Network* (TIN) [DPM99]. Regular grids can be encoded in very compact data structures, since only the elevation values need to be stored. TINs, on the other hand, better adapt to the shape of the terrain, since their vertices are irregularly and adaptively sampled. A geometry-based description, such as an RSG or a TIN, provides an accurate representation of a terrain, but fails in capturing its morphological structure defined by critical points, like pits, peaks or passes, and integral lines, like ridges or valleys.

There has been a lot of research in the last decades focusing on extracting critical features (points, lines or regions) from images or terrain data described by an RSG, or a TIN. More recent works in computational geometry concentrate on representing the morphology of terrains through a decomposition of the terrain surface into regions bounded by critical points (minima, maxima, saddle points) and integral lines [EHZ01]. These techniques are rooted in Morse theory and try to simulate the decomposition of a terrain induced by C^2-differentiable Morse functions in the discrete case.

A hierarchical representation of the terrain morphology is critical for interactive analysis and exploration of a terrain in order to maintain and analyze characteristic features at different levels of resolution. Current multi-resolution terrain models are just based on a geometric simplification process applied to a TIN describing a terrain at full resolution. In [DDM*03a], we have defined a multi-resolution TIN which encompasses the morphology of the terrain. This is

† corresponding author: papaleo@disi.unige.it

achieved through an algorithm for simplifying a constrained TIN through vertex removal, where the constraints are represented by the polygonal edges which describe the integral lines connecting the critical points at different resolutions. In such algorithm, however, the critical points have been maintained at the different levels of detail, so as to keep the morphology of the contour lines. In [BEHP04], a multi-resolution representation of a triangulated terrain has been proposed based on a generalization of the morphological terrain and on a re-meshing of the regions in the underlying triangle mesh at each generalization operation. In [BPH05], the hierarchical structure has been modified to reduce dependencies among generalization steps.

In our work, we consider a combined multi-resolution terrain representation based on a multi-resolution constrained TIN, and on a multi-resolution structural description of the terrain morphology. The multi-resolution constrained TIN is generated by simplifying the TIN following the generalization process which guides the simplification of the underlying morphology and, thus, the hierarchical morphological representation. Here, we focus on the multi-resolution morphological model. More precisely, we consider the surface network, which is a graph-based representation of the terrain morphology, and we discuss a generalization operator for simplifying such network. Based on such operator, we have developed a hierarchical representation of a surface network that we call a *Multiresolution Surface Network* (MSN). An *MSN* consists of a surface network representing the terrain morphology at a coarse resolution, and of a collection of refinement modifications, which reverse the generalization operators used in simplification, organized as a Directed Acyclic Graph (DAG). Variable-resolution surface networks can be extracted from an *MSN* through a simple DAG traversal.

The remainder of this paper is organized as follows. Section 2 reviews some background notions on morphological representations based on Morse theory. Section 3 reviews different approaches for computing discrete approximations of the terrain morphology by applying Morse theory. Section 4 formalizes a generalization operator for simplifying a surface network. Section 5 introduces the multi-resolution surface network (MSN). In Section 6, some concluding remarks are drawn.

2. Background Notions

In this Section, we introduce some basic notions on Morse theory and on the decomposition of the domain of a scalar field induced by a Morse function. For simplicity, we report the definitions only for the case of 2D scalar fields defined over a bounded region in the plane, but they extend to arbitrary dimensional scalar field defined over a manifold in $\mathbb{R}^d$.

Morse theory is a powerful tool to capture the topological structure of a scalar field. Let f be a C^2-differentiable real-valued function defined over a domain $D \subseteq \mathbb{R}^2$. A point $p \in \mathbb{R}^2$ is a *critical point* of f if and only if the Gradient of f vanishes at p. A function f is said to be a *Morse function* if all its critical points are non-degenerate. This implies that the critical points of a Morse function are isolated. An *integral line* of a function f is a maximal path which is everywhere tangent to the gradient vector field. An integral line is emanating from a critical point or from the boundary of D, and it reaches another critical point or the boundary of D. An integral line which connects a maximum to a saddle or a minimum to a saddle is called a *separatrix line*.

The integral lines that converge to a maximum, a saddle and a minimum form a 2-dimensional, 1-dimensional and 0-dimensional region, respectively, and they are called *stable manifolds*. The integral lines that originate from a minimum, a saddle and a maximum form a 2-dimensional, 1-dimensional and 0-dimensional region, respectively, and they are called *unstable manifolds*. The stable (unstable) manifolds are pair-wise disjoint and decompose surface S into open cells which form a complex, since the boundary of every cell is the union of lower-dimensional cells. Such complexes are called *stable* and *unstable Morse complexes*, respectively. Figure 1(a) shows an example of a decomposition of the domain of a scalar field into an unstable Morse complex.

A Morse function f is a *Morse-Smale function* when the stable and the unstable manifolds intersect only transversally. In two dimensions, this means that the stable and unstable 1-manifold cross when they intersect, and the crossing point is a saddle point.
A *Morse-Smale complex* is the complex defined by the intersection of the stable and unstable Morse complexes. The 1-skeleton of a Morse-Smale complex consists of the critical points and the separatrix lines joining them, and it is called a *critical net* (see Figure 1 (b)). Figure 1(b) shows an example of a Morse-Smale complex for the same function as in Figure 1(a).

A combinatorial representation of the critical net in the case of 2D scalar fields, widely used in Geographic Information Systems (GISs), is provided by the *surface network* [Pfa76, SW04]. A surface network is a planar graph, in which the nodes correspond to the critical points, and the arcs to the separatrix lines connecting them. There exists an arc between a pair of nodes in the surface network if the two corresponding critical points are connected by a separatrix line in the critical net.

3. Computing approximations of Morse and Morse-Smale complexes

Several algorithms have been proposed in the literature for decomposing the domain of a scalar field f into an approximation of a Morse complex, or of a Morse-Smale complex. Such an approximation is obtained either by fitting a C^1- or

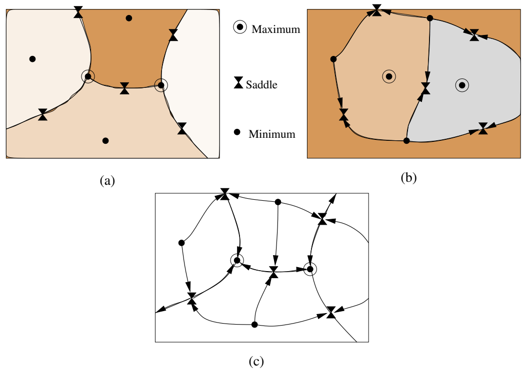

Figure 1: *(a) An example of an unstable Morse complexes (the 2-cells correspond to the minima). (b) The Morse-Smale complex. Its 1-skeleton is the critical net.*

C^2-differentiable surface on a terrain dataset, or by simulating a Morse-Smale complex, or a Morse complex in the discrete case by inferring properties from the C^2-differentiable case. Several recent algorithms working on TINs use this latter approach. Their assumption is that no two adjacent vertices in the TIN have the same elevation. This ensures that the critical points are isolated, as in the case of C^2-differentiable Morse functions.

All the algorithms proposed in the literature, with the exception of the one in [EHNP03], have been developed for 2D scalar fields. Almost all of them use a *boundary-based* approach, in the sense that they extract an approximation of the critical net, by computing the critical points and then tracing the integral lines starting from saddle points and converging to minima and maxima.

Most algorithms working on TINs [TIKU95, BS98, EHZ01, BEHP03, Pas04] detect first the critical points by comparing the elevation values at each vertex p with the elevations at the vertices adjacent to p on the TIN, and then compute the 1-cells of the complex by starting from the saddle points, and tracing two paths of steepest descent and two paths of steepest ascent on the underlying triangle mesh which stop at minima and maxima, respectively. The algorithms in [TIKU95, EHZ01, BS98] compute paths along the edges of the triangle mesh by selecting either the vertex of highest (or lowest) elevation at each step [BS98, TIKU95], or the steepest ascending or descending edge at each step [EHZ01]. The algorithms in [BEHP03, Pas04] estimate the gradient along edges and triangles, and compute the ascending and descending paths by also cutting triangles in order to follow the actual paths of steepest ascent, or descent. The algorithms in [BPS98, SW04, Sch05] compute the arcs of the critical net from a 2D regular model, through a technique conceptually very similar to the one used for TINs. All three algorithms fit a surface with a certain degree of continuity to the input data set in order to extract the critical points. All

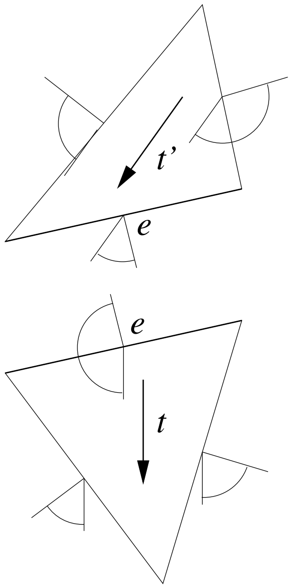

Figure 2: *Triangle t is adjacent to t', e is best exit for t' and best entrance for t. t is added to the region of t'*

these approaches try to enforce the Smale condition in the discrete case, by avoiding to connect two saddle points.

In [DDM*03a, DDM03b], we have proposed an entirely different approach, that we call *region-based*. It consists of computing the stable and unstable Morse complexes from a TIN. The only assumption is that the piecewise linear function defining the TIN is a discrete Morse function. Recall that the unstable (stable) manifold of a point p for a Morse function f is the set of points q such all that the ascending (descending) integral lines from q reach p. Our algorithms simulate this definition in the discrete case. First all minima and maxima are identified, and then the ascending and descending complexes are computed independently by applying a region-growing approach. The first algorithm [DDM03b] computes the stable complex starting from a minimum m and initializing the 2-cell which corresponds to m with the triangles incident at it. At a generic step, the 2-cell associated with m is extended by adding a new triangle t sharing an edge e with the cell, provided that the vertex of t not bounding e has an elevation value highest than that of the extreme vertices of e. In [DDM*03a], the same region-growing approach is applied, but the gradient for each triangle t in the TIN is computed, and the angles between the normal vector at each edge of t and the gradient are evaluated. The edge e of t corresponding to the largest angle is marked as *exit*, the one corresponding to the smallest angle is marked as *entrance*. Thus, the new triangle t for extending a 2-cell is selected as a triangle sharing an edge e with the cell such that e is an entrance for t and an exit for the triangle t' in the cell sharing edge e with t (see Figure 2).

The unstable complex is computed in a completely symmetric way. The intersection of the stable and unstable Morse complexes then approximates the Morse-Smale complex, if the boundary of the regions in the two complexes intersect at a saddle point. Saddle points are extracted as the intersection of the two complexes. Although proposed for TINs, the approach can be extended to 3D scalar fields whose domain is discretized as a tetrahedral mesh.

Watershed algorithms developed for image segmentation and applied to terrains can also be viewed as region-based methods for computing the stable and unstable Morse complexes [Mey94, VS91, MW99] when the underlying field function satisfies the discrete Morse property, discussed above.

4. Generalization of Morse-Smale complexes

Two major issues arise when computing a representation of a scalar field as a Morse, or a Morse-Smale complex. The first issue is the over-segmentation due to the presence of noise in the data sets. To this aim, *generalization algorithms* have been developed by several authors to locally simplify the structure of a Morse-Smale complex [Wol04, EHZ01, BEHP04, TIKU95, Tak04, GNP*05]. The second issue is related to the large size and complexity of available scientific data sets. Thus, a multi-resolution representation is crucial for an interactive exploration of such data sets. There exist just a few proposals in the literature for multi-resolution representations for 2D scalar fields [DDM*03a, BEHP04, BPH05].

The generalization of a Morse-Smale complex for a two-dimensional scalar field consists of collapsing a maximum-saddle pair into a maximum, or a minimum-saddle pair into a minimum, so as to maintain the consistency of the underlying complex. Usually, this operation is viewed as the *cancellation* of a pair of critical points, namely, a maximum and a saddle or a minimum and a saddle. A cancellation simulates the smoothing of the scalar field by modifying the gradient flows around two critical points.

We have formalized a cancellation in terms of the combinatorial representation of the critical net, defined by the surface network, as described below (see [DDPV05] for more details). Let $S_N = (C,A)$ denote the surface network for a 2D scalar field. Let p and s be two critical points (i.e., two nodes in S_N such that arc $(p,s) \in A$, and p is a minimum (maximum) and s is a saddle. We call the *Influence set* I^+ of (p,s) the collection of arcs $a_1,\dots,a_k$ in A which are incident either in p or in s.

$$I^+ = \{e \equiv (t,v) \in A, |\{p,s\} \cap \{t,v\} \neq \emptyset\}$$

We call *relevant saddles* those saddles, different from s, which are connected to p through an arc belonging to I^+. Let R_s denote the set of relevant saddles with respect to the pair (p,s).

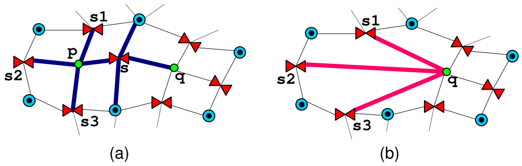

Figure 3: *(a) A surface network $S_N = (C,A)$. The arcs in I^+ are highlighted in dark blue. (b) The surface network $S'_N = (C',A')$ obtained from S_N by cancellation of saddle s and minimum p. The arcs in I^- are highlighted in purple.*

$$R_s = \{s_i \in C, s_i \neq s | \exists e \in I^+ \wedge e \equiv (s_i,p)\}$$

Moreover, because of the definition of surface network, if p is a minimum (maximum) there must exist exactly one other minimum (maximum) q, different from p connected to s through an arc in I^+. Let I^- denote the set of arcs connecting q with all the relevant saddles. Thus, $I^- = \{(q,s_i) | s_i \in R_s\}$ and, obviously, $I^- \cap A = \emptyset$. The generalization transformation (*cancellation*) on a surface network $S_N = (C,A)$ is thus defined as follows:

$$C = C \setminus \{p,s\}, A = (A \setminus I^+) \cup I^-$$

In other words, points p and s are removed from C and the arcs in I^- are replaced with I^+ in S_N. Figure 3 (a) shows an example of a surface network $S_N = (C,A)$. The arcs in I^+ are highlighted in dark blue. Figure 3 (b) illustrates the surface network $S'_N = (C',A')$ obtained from S by canceling the points s and p, where s is a saddle and p is a minimum. The arcs in I^- are highlighted in purple.

Generalizations (or contractions) of surface networks have to be such that the resulting surface network should always be topologically consistent. The main difference among the methods proposed in the literature is in the way pairs of critical points to be canceled are selected. In [Wol04], a minimum (maximum) p is chosen for cancellation together with its lowest (highest) adjacent saddle s. The order in which the minima and maxima are chosen for the cancellation is not specified. In [EHZ01], and in [BEHP03], a saddle s is selected together with its adjacent maximum at lower elevation, or its adjacent minimum at higher elevation. The order in which the pairs of points are canceled is determined based on the notion of *persistence* (see [BEHP03] for more details on persistence). In the generalization algorithm proposed in [Tak04], a pair of adjacent critical points p and s is chosen in such a way that the difference in elevation between p and s is minimal among all (unsigned) differences in elevation between a saddle and an adjacent minimum, or a saddle and an adjacent maximum (see Figure 3).

5. Multi-resolution Surface Networks

Let us consider a sequence of legal generalizations applied to the surface network $S_N = (C, A)$ at the maximum resolution. This sequence produces a surface network at the coarsest resolution, the we call the *base network*. We can invert the cancellation sequence, by considering the base network plus a sequence of refinements. A *refinement* is the inverse operation with respect to a cancellation. We observe that some refinements do not have to be necessarily applied in the same order as in the sequence. We can thus define a *dependency relations* among refinements. Intuitively, two refinements are considered to be independent if they do not affect the same portion of surface network. If u and w are two independent refinements, then u can be applied before w, or w before u. Thus, a multi-resolution representation for a surface network encodes the surface network at the coarsest resolution, plus the a collection of refinements, reversing the cancellation sequence, and a dependency relation among them. The hierarchical Morse-Smale complexes introduced in [BEHP04, BPH05] can be seen as an instance of such representation.

A cancellation applied to a surface network $S_N = (C, A)$ can be expressed as a pair (I^+, I^-), as explained in the previous section. We denote with $S'_N = (C', A')$ the surface network obtained from S_N by replacing I^+ with I^-. Then, the inverse refinement transformation, applied to S'_N consists of replacing the arcs in I^- with those in I^+, thus yielding network S_N as result. We call the pair $u = (I^-, I^+)$ a *refinement update*. A refinement update $u = (I^-, I^+)$ can be applied to a surface network $S_N = (C, A)$, if and only if $I^- \subset A$ and also $u' = (I^+, I^-)$ satisfies the requirements to be a feasible cancellation transformation (as defined in Section 4). Thus, we can define a *dependency relation* between pairs of refinement updates $u_1 = (I_1^-, I_1^+)$ and $u_2 = (I_2^-, I_2^+)$ as follows: u_2 *directly depends* on u_1 if and only if u_2 removes some of the arcs inserted by u_1, thus if and only if $I_1^+ \cap I_2^- \neq \emptyset$. We denote the *direct dependency relation* as $\prec$.

Figure 4 shows a surface network at the coarsest resolution (base network) and the refinement transformations with their dependency relations. The updates $1 \equiv u_1 = (I_1^-, I_1^+)$ and $2 \equiv u_2 = (I_2^-, I_2^+)$ involving pairs (s_1, p_1) and (s_2, p_2), respectively, are independent. Note that $I_1^- \cap I_2^+ = \emptyset$ and $I_2^- \cap I_1^+ = \emptyset$. The refinement update $3 \equiv u_3 = (I_3^-, I_3^+)$ depends on both the updates 1 and 2, since $I_3^- \cap I_1^+ = p_1$ and $I_3^- \cap I_2^+ = p_2$.

The transitive closure of relation $\prec$ can be shown to be a partial order. Thus, we call the pair $M = (U, \prec)$ a *Multi-resolution Surface Network (MSN)*. In Figure 4 an example of an *MSN* is depicted. Note that the direct dependency relation is represented as a Directed Acyclic Graph (DAG). The refinements belonging to any subset U' on the set of refinements U, that is closed with respect to the partial order (i.e. such that, for every refinement $u' \in U'$ also the refinements

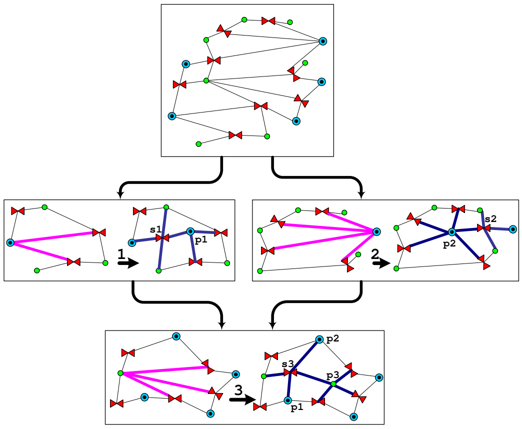

Figure 4: *The DAG of the refinement updates. On the top of the picture the base network is depicted. The refinement updates* 1 *and* 2 *are independent, while the update* 3 *depends on both* 1 *and* 2*.*

preceding u' belong to U') can be applied to the base network S_B in any total order that extends the partial order, thus producing a surface network at an intermediate resolution.

It can be easily seen that any extracted surface network is a planar graph. The base network S_B is a planar graph, since each refinement removes a set of arcs I^- which are all internal to a cycle defined by the relevant saddles and the new set of arcs I^+ are internal to the same cycle.

The basic operation that we need to perform on a multi-resolution surface network consists of extracting a surface network from an *MSN* satisfying some application-dependent requirements based on the level of detail (LOD), such as the density of the critical points, the difference in elevation of pairs of removable points, etc. The LOD criterion can be uniform, or variable in space. Such operation is called *selective refinement*.

A selective refinement algorithm traverses the partially ordered set U of updates and constructs a closed subset U' of updates that, when applied to the base network S_B, gives the network which is the answer to the selective refinement query. Figure 5 shows examples of networks extracted from a triangulated terrain. Figure 5 (a) illustrates the coarsest surface network, Figure 5 (b) and Figure 5 (c) represent intermediate refined surface networks. Finally, Figure 5 (d) illustrates the surface network at full resolution.

A direct encoding of an *MSN* by storing, for each update $u = (I^-, I^+)$, the set of arcs in I^- and I^+ can be inefficient in term of space. Thus, we have developed a compact representation that we describe below. The direct dependency relation is encoded as a DAG in which the nodes correspond to refinements and the arcs describe the direct dependency relation. Updates are described not as collections of arcs, but procedurally. The encoded information must be sufficient to

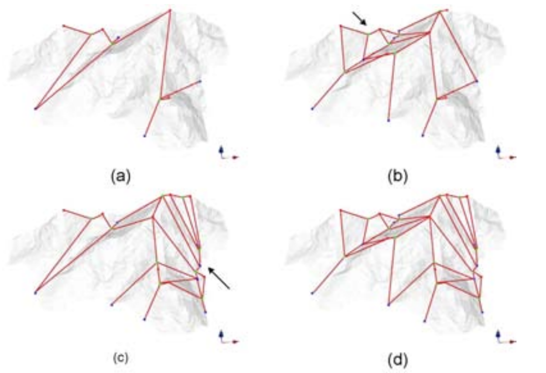

Figure 5: *(a) The initial triangulated terrain and the coarsest surface network. (b)-(c) Two intermediate surface networks. (d) The surface network at full resolution.*

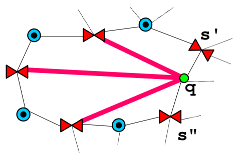

Figure 6: *The arcs in I^- are marked in purple. q is a minimum and s', s'' are saddles.*

perform both cancellations and refinement on any currently extracted surface network. A refinement is required when extracting a network at some intermediate resolution by top-down traversal of the DAG, while we need to perform cancellations to coarsen locally any extracted network.

The cancellation transformation is entirely specified by the pair of critical points (s,p) removed, where s is a saddle point and p a minimum (maximum). To perform the inverse refinement transformation $u = (I^-, I^+)$ we need to specify the two critical points p and s inserted by the refinement (coordinates and field value), and an implicit description of I^-. This is obtained by specifying:

- the critical point q which is the extreme vertex of every arc in I^-.
- ordered pair of relevant saddle points (s', s'') satisfying the following condition:
  - there exist two arcs (s', q) and (s'', q) which are incident in q.
  - I^- is the set of arcs incident in q which are between arcs (s', q) and (s'', q) by considering the arcs incident in q in counterclockwise order around q (see Figure 6).

Note that I^+ is then completely defined since the extreme nodes of the arcs in I^- which are different from q define the relevant saddles (see Section 4). Thus, an update is encoded in 24 bytes since we assume to store the coordinate, the field value, or a pointer in 4 bytes.

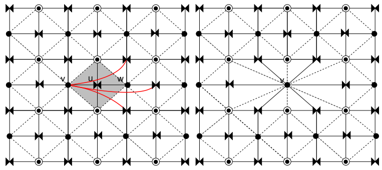

Figure 7: *(a)The diamond in gray will be deleted. (b)The resultant diamonds after the cancellation.*

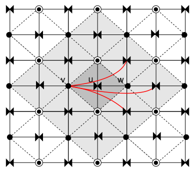

Figure 8: *Region of influence in [BEHP04]*

The model proposed in [BEHP04] is based on the simplification of the critical net instead of the surface network, and thus is a combined geometrical and topological hierarchy, while an *MSN* is a strictly combinatorial one. When a pair of critical points p and s are canceled, they are removed from the mesh through gradient smoothing that introduces an error which is bounded by half the persistence of (p,s). The regions in the new Morse-Smale complex are smoothed and the paths joining the relevant saddles to the minimum or maximum, on which p and s are collapsed, are recomputed using the new geometry. The central element of this data structure is the diamond. Each diamond is centered in a saddle and consists of a quadrangle whose vertices alternate between minima and maxima around the saddle (see Figure 7). A cancellation corresponds to removing a diamond and re-connecting its neighbors. Two cancellations are considered independent when their diamonds share no vertex. This rule is over constraining, since some diamonds that share a vertex with the removed diamond are not actually affected by the cancellation (see Figure 8). The work in [BPH05] improves over the previous hierarchical representation by reducing the number of dependencies. It can be easily seen that the dependency relation in [BPH05] is the same as in an *MSN*, while the data structure combines a tree representation of the cancellation operation with a dependency graph, but its implementation is not specified. The data structure corresponds to a direct implementation of the dependency relation in a MSN as a DAG and to an explicit encoding of the updates by specifying sets I^+ and I^- as collection of edges. The major advantage of the compact data structure we have proposed here is its efficiency in term of space requirements.

6. Concluding Remarks

We have considered the problem of representing the morphology of a scalar field at different resolutions. The terrain morphology is described through a Morse-Smale decomposition of its domain, which is abstracted in the surface network. We have then formalized the generalization operator on a surface network and defined a hierarchical representation in the form of a multi-resolution surface network (*MSN*) to be combined with a multi-resolution geometry-based terrain model, which encompasses the morphology simplification at different resolutions. We have shown an efficient implementation of an *MSN* and how variable resolution representations can be extracted from it.

Further development of this work involve developing efficient techniques for computing Morse complexes for 3D and 4D scalar fields, and investigate multi-resolution morphological representations of 3D scalar fields based on Morse decompositions. To this aim, we are currently extending the algorithm in [DDM*03a] to the 3D case.

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