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Celtic Knots Colorization based on Color Harmony Principles

Caroline Larboulette

Universidad Rey Juan Carlos, Madrid, Spain

Abstract

This paper proposes two simple and powerful algorithms to automatically paint Celtic knots with aesthetic colors. The shape of the knot is generated from its dual graph as presented in [KC03]. The first technique uses rules derived from two-colors harmony studies in a Color Order System to select harmonious color pairs. We show that it can efficiently reproduce color combinations utilized in ancient and modern Celtic design. The second technique aims at creating knots with a rainbow *type colorization that can be seen in modern Celtic art. The user controls a few parameters like the number of desired different hues, or the average brightness and saturation expected, by defining an ellipse in the color space. The program then accordingly selects a series of colors. In both cases, we apply rules issued from prior Color Science studies.*

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Color, shading, shadowing, and texture

1. Introduction

Celtic design is an ancient artwork which seems to have origins as early as 500 B.C. The beauty and mystery of Celtic knots make it a quite inspiring source for modern artists, who try to understand traditional Celtic knot construction in order to be able to create new and different modern knots. Although the construction rules remain the same, the knots generated look quite different from those that can be found in ancient manuscripts. Figures 4, 5 and 14 show some examples.

According to the survey we proposed to 32 observers, there are two attributes responsible for the aesthetic value of a Celtic knot: its *shape* and its *color*. While 81% of the observers mentioned one or both of those attributes, only 16% considered the color was the only important characteristic and 22% the shape only.

Previous work in Computer Graphics propose efficient algorithms to generate correct shapes for various kinds of knots: regular [Gla99, KC03], cantors [Bro05] or fractals [Bro06]. With those algorithms, it is possible to create or recreate almost any possible knot, also with images embedded. However, no research has been done with respect to the colorization of those knots.

Finding pleasing colors to paint an artwork requires an aesthetic measure to select the colors. In the literature, this measure of aesthetics for colors is often referred as color harmony. The definition of color harmony often adopted and that we will adopt in this paper is the one given by Judd and Wyszecki [JW75]: "when two or more colors seen in neighboring areas produce a *pleasing* effect, they are said to produce a color harmony."

Our goal in this paper is thus to propose a way to choose harmonious colors to paint Celtic knots. Although color harmony is often studied as an objective problem, we think that the pleasing effect generated (or not) on a person depends on the use that is made of the colors. For example, an harmonious color set to paint a kitchen might not be an harmonious color set to paint Celtic knots. In our case, Celtic knots are 2D designs where different colors are present next to each other. Research from Color Science, where color matching is done by showing two color samples *next to each other* to a number of people, can thus be directly applied (as opposition to e.g. architecture where the objects occupy a 3D space and are made of different materials).

In the remaining of this paper, we first introduce some related work in Color Science and Computer Graphics in section 2. Section 3 then describes the main characteristics of ancient and modern Celtic knots. Section 4 presents our algorithms while section 5 presents and discusses a few results. Finally, section 6 concludes and proposes some possible future work.

2. Related Work

2.1. Color Order Systems

One common way to pick up colors to paint objects is to use a Color Order System (COS). Every COS arranges colors according to their major perceptual attributes such as their *hue*, *brightness* and *saturation*. A COS is usually presented under the form of an atlas. A typical page presents a single hue, which is derived with increasing brightness and saturation. An example of such a page is shown on figure 1.

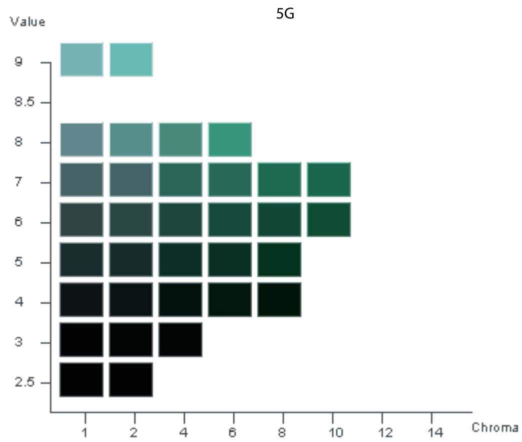

Figure 1: *An illustration of the Munsell Hue* 5*G page of the Munsell Book of Color [Mun15, Mun76]. The Munsell Value (brightness) increases on the vertical axis, while the Munsell Chroma (saturation) increases on the horizontal axis.*

The most common COS used in industry is the Munsell Book of Color [Mun15,Mun76]. It orders the colors by mean of their perceptual difference in hue, saturation and brightness. It is thus a very appropriate choice for our algorithms. Although the Coloroid system [Nem80, Nem87] might appear as a better choice because it orders colors by mean of their aesthetic differences, we found it not practical to use with the color harmony principles we will detail in the next subsection.

Munsell Book of Color
Colors in this system are arranged such that the perceptual difference between any two neighboring colors is nearly constant in each of the three dimensions, Munsell Hue, Munsell Value and Munsell Chroma.

Munsell's practical principles [Mun05, Mun21] of color harmony are based on the idea that colors can harmonize only when they are located on a specific path in his color space. These paths include:

1. The grey scale (vertical Value axis);
2. Colors of the same Munsell Hue and the same Munsell Chroma (any vertical line);
3. Complementary colors of the same Value and the same Chroma;
4. Colors of *diminishing sequences*, in which each color is dropped down one step in Value as Chroma goes down one step, and so is Hue;
5. Colors on an *elliptical path* in the Munsell color space (i.e. colors on a 3*D* ellipse).

The only drawback of the Munsell color space is that it is made from existing pigments and is therefore only valid for one illuminant and is discontinuous. However, as the colors are ordered by perceptual differences, it is easy to order the Hue pages in a 3*D* cylindrical space as you can see on figure 2. The other direct consequence is that colors with the same brightness and the same saturation are placed on circles centered around the brightness axis.

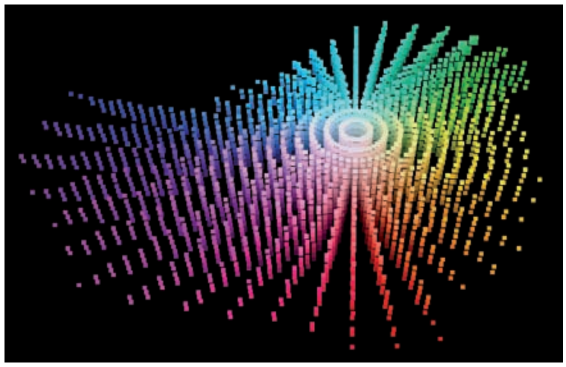

Figure 2: 3*D plot of the Munsell Color Order System.*

2.2. Color Harmony

Recent studies [CO01, OL06] on color harmony show some interesting results concerning two-colors combinations. The authors presented 17 observers with 1431 color pairs of varying hue, chroma and brightness. The color pairs were systematically generated using 54 colors from the CIELAB color space. The samples were showed side by side on a medium grey background on a CRT monitor in a darkened room. The observers were asked to rate the combinations on a 10-categories scale varying from "extremely harmonious" to "extremely disharmonious". The experimental results showed a general pattern of two-colors harmony from which principles for creating harmony were derived:

6. **Equal-hue and equal-chroma**. Any two colors varying only in lightness tend to appear harmonious when combined together. In Munsell color space, this is equivalent to selecting two colors on the same vertical.
7. **High lightness**. The higher the lightness value of each constituent color in a color pair, the more likely it is that they will appear harmonious.
8. **Unequal lightness**. Small lightness variations (typically less than 15 units of CIELAB color difference) between

the constituent colors in a color pair may reduce the harmony of that pair.

9. **Hue effect**. Among various hues, blue is the most likely to create harmony in a two-colors combination, red is the least likely to do so. Bright yellows more often create harmony in two-colors combinations than dark yellows.
10. **Contrasting Hue**. Colors contrasting in hue are more harmonious.

In addition to those results, another hue effect was discovered. Although it was judged not acceptable in providing an accurate prediction of color harmony, we thought it was an interesting result, especially applicable to Celtic knotwork. Figure 3 shows that most color pairs (except dark colors) will harmonize if their hue angle is around 90 and 270 degrees (rule $n^o 11$).

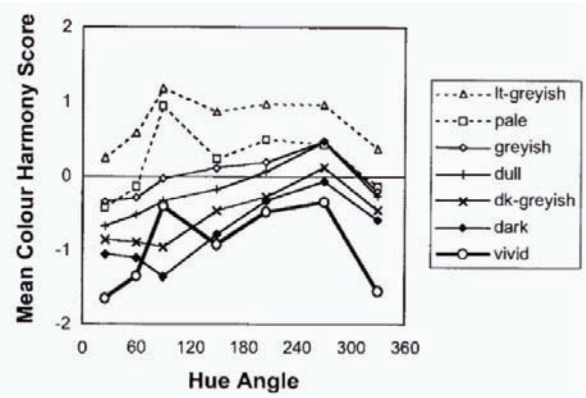

Figure 3: *Color harmony scores for color pairs that contain a color of a specific hue. Note that for most colors (except dark ones), the best scores are obtained at hue angles of* 90 *degrees and* 270 *degrees. Graph extracted from [OL06].*

Finally, the only work we are aware of in Computer Science is the work of Neumann et al. [NNN05]. The authors propose a way of creating harmonic sets of 2 or 3 colors using the Coloroid system. The mathematical formulas used in the paper have been derived from observations of people who all belonged to the same ethnic group (Hungary). The outcome of this work is that colors with different hues will harmonize if they are distant of ± 12, 35, 130 and 180 degrees in Hue angle in the Coloroid system. Those results are however not directly transposable to another Color Order System and were applied to $3D$ architectural design while we aim at $2D$ colorization applications.

3. Celtic Knots under Study

The most famous testimony of Celtic knotwork is the Book of Kells produced by Celtic monks around A.D. 800. The knots represented in that book are often painted in vivid colors, with rather low brightness and huge contrast in hue. As we can see on figure 4, knots are painted with one color per thread and composed of pairs of contrasting colors like green and yellow, blue and red, violet and yellow. Although one could think at first glance that the colors used have been chosen because they were the only existing pigments, we will show with our algorithms that the color pairs used followed two-colors harmony rules (at least 10 colors were used in the book, hence $C_{10}^2 = 45$ possible combinations).

Figure 4: *A close-up view of the top part of a page of the Book of Kells. Note the color composition: green with yellow, blue with red and purple with yellow or green. All the colors are very saturated and quite dark.*

While ancient artwork was rather using a single color per thread, nowadays artwork tend to mix colors a lot more. This is probably due to the existence of color pencils which are easier to manipulate than pen and ink. Figures 5 and 14 show a few modern Celtic knots designed by hand. In many of the modern knots we studied, the colors used were of type *rainbow*, i.e. smoothly varying through different hues.

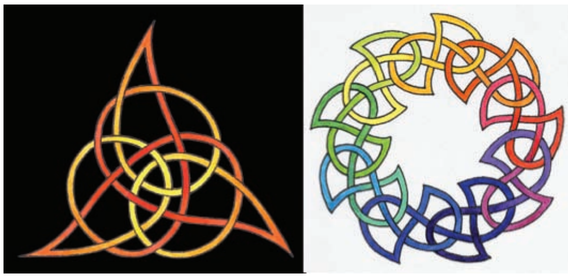

Figure 5: *Two examples of modern Celtic artwork. In both cases, the artist used rainbow colors to paint the knot. Note that the threads are colorized in a symmetric manner. Drawings courtesy of Sasha Kopf.*

Therefore, we propose two different techniques for colorizing Celtic knots. The first one is based on two-colors harmony and aims at picking different colors that harmonize to paint the different threads of a knot. This technique can be used to replicate ancient artwork like what can be found in the well-known Book of Kells. The second algorithm, which we call *rainbow* algorithm, aims at painting the knots with gradients of colors. It picks a whole set of neighboring colors and can be used to paint more modern Celtic artwork. As a simplification, we also propose the *shadow* algorithm which only uses a single hue and a single chroma but varying brightness. The resulting effect looks like shadows have been applied to the knot threads.

4. Colorization Algorithms

4.1. Our Framework

To generate the shape of our knots, we implemented the algorithm presented in [KC03]. This algorithm is efficient and easy to use to design any type of knots shapes. The threads are calculated using Hermite Splines with a similar smoothing factor as the one presented in the original paper. The great advantage of using splines is that it gives us a parameterization of each thread chunk. A chunk is the piece of parametric curve that goes from one intersection to the next intersection. The t parameter along the spline is used to colorize the *shadow* and *rainbow* knots. A mapping between the spline parameter t and the aesthetic path shape parameter is created.

The Munsell COS data originally given in the CIE xyY space was converted to RGB for OpenGL rendering in our program. This conversion was achieved in two steps, using the following formula:

step1: CIE xyY -> CIE XYZ

$$X = (xY)/y; Y = Y; Z = ((1-x-y)Y)/y$$
$$\text{if } y = 0, X = Y = Z = 0$$

step2: CIE XYZ -> RGB with a gamma of 2.4

$$r = 0.032406 * X - 0.015372 * Y - 0.004986 * Z$$
$$g = -0.009689 * X + 0.018758 * Y + 0.000415 * Z$$
$$b = 0.000557 * X - 0.002040 * Y + 0.010570 * Z$$

$$C = \begin{cases} 12.92 * c & \text{if } c \leq 0.0031308, \\ 1.055 * c^{(1/2.4)} - 0.055 & \text{if } c > 0.0031308 \end{cases}$$

With $C = R$, G or B and $c = r$, g or b. In addition, we clamp the colors between 0 and 1.

4.2. Knots with one Color per Thread

There are two main possibilities when selecting 2 different colors to create a color harmony pair: choosing two colors with the same hue, or two different ones. Although Ou & Luo's paper [OL06] concludes that we can only be sure to create a harmony when we select two colors having the same hue, in Celtic design, the common practice is to select colors having different hues. The same study shows that if one has to choose colors with different hues, then the best harmony pair is created with colors having a ± 90 degrees difference in hue angle (see section 2.2).

As this does not apply to dark colors, by default, if a color is dark (Munsell Value < 3), we apply the *equal-hue and equal-chroma* principle (rule $n^o 6$) . This consists in selecting a second color with the same hue and same chroma, but higher brightness, the higher the better (this follows the principle of *unequal lightness* – rule $n^o 8$ – presented in section 2.2). In our implementation, we increase the brightness of the input color by 6 Munsell Value units to create the second color. Figure 6 shows some examples of results.

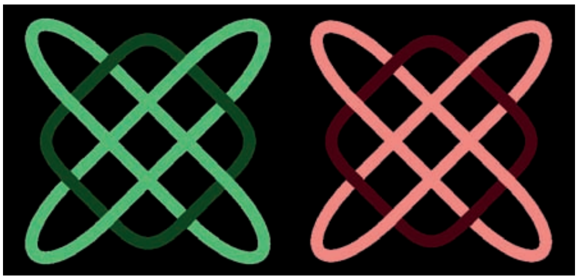

Figure 6: *Two Celtic knots colorized with one color per thread and following the* equal-hue and equal-chroma *principle (rules $n^o 6$ and 8).*

However, most colors in Celtic design are dark. Because of the limited pigments available, dark colors with contrasting hues had to be combined to create aesthetic designs. It happens that the combinations used in the Book of Kells follow the rule of the ± 90 degrees difference in hue angle (rule $n^o 11$). We thus applied this rule in our algorithm. Of course, to ensure a maximum harmony, if the hues do contrast, the brightness and chroma have to be kept constant (rule $n^o 3$). Figure 7 shows the 3 possible knots for the same dark red input color. From left to right, we can see the *equal-hue and equal-chroma* picture, followed by the *equal-chroma, equal-brightness and* 90 *degrees in hue* knot, and finally the 270 *degrees in hue* knot.

4.3. Knots Colored with Gradients of Colors

4.3.1. Shadow Knots

The *shadow* knots are based on the aesthetic rules of *equal-hue and equal-chroma* and *unequal lightness* like previously. Those rules combined say that two-colors can harmonize if they have the same hue, same chroma, and a high difference in brightness. In Munsell COS, this corresponds to selecting a second color on the same vertical as the first one. In addition, in his principles of aesthetics, Munsell introduced the notion of *path*. The general path that should be used is the ellipse (rule $n^o 5$), and this is the purpose of the next section (*rainbow* knots). However, we felt the necessity of having a simplified version of the ellipse parametric curve. For *shadow* knots, we thus simply use the straight line between the two colors in the set of harmonious colors as the parametric curve (rule $n^o 2$).

For implementation purposes, we defined a color to be

Figure 7: *3 knots with one color per thread and the same dark red input color. The first knot applies the rule of* equal-hue and equal-chroma *while the second and third knot respectively use a hue selected at* 90 *degrees and* 270 *degrees in Munsell COS.*

bright when its Munsell Value is greater than 5. If the selected color is dark, the second color selected is a brighter one (practically, we choose a color 5 Munsell Value units brighter). Similarly, if the color is bright, we select a darker one. The first chosen color is applied at $t = 0$ and the second color is applied at $t = 1$. This order is reversed at each interleaving. As the parameter t varies, we go from the first selected color to the second color of the pair, on a straight vertical segment in Munsell color space. As a difference of 5 units in Munsell Value means we will only go though 6 colors while painting the whole knot, we have to *interpolate* the colors in the RGB space. For this purpose, we linearly interpolate the RGB colors of the 8 nodes that surround our current color position in the color space. The result of the interpolation is shown on figure 8.

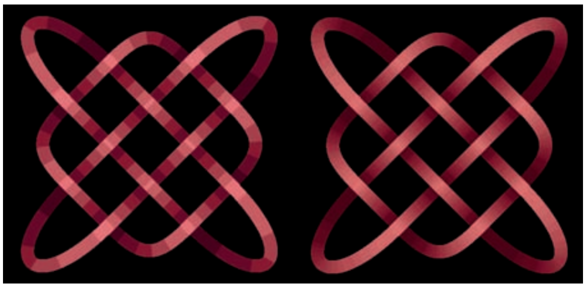

Figure 8: *Illustration of the interpolation of RGB colors on a pink Celtic knot. On the left, there is no interpolation, we can see the* 6 *different colors. On the right, we linearly interpolate the colors of the* 8 *surrounding nodes.*

4.3.2. Rainbow Knots

Rainbow colored knots illustrate Munsell's principle of *elliptic path* (rule n^o5). In order to make smooth color changes along one thread of a knot, we change its color as we follow the parametric curve (ellipse) defined by the user in the COS. There is a direct mapping between one chunk of a thread and the path on the ellipse. We start at some point on the ellipse at $t = 0$ (first node of a chunk) and we reach this same point by going through the entire ellipse at $t = 1$ (second node of the chunk). The user can directly control all ellipse parameters: the position of the center, the length of its axis, the plane in which it deforms and its eccentricity. Of course, care has to be taken so that the ellipse remains in the color space. Figure 9 shows such an ellipse in Munsell COS.

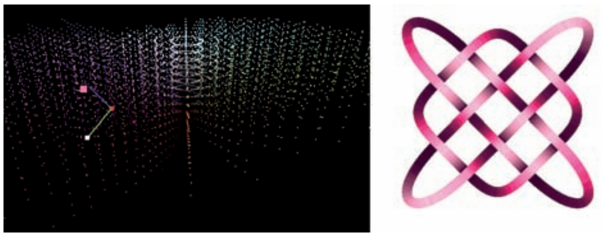

Figure 9: *On the left, you can see the position of the ellipse in the Munsell color space. The red dot is the center of the ellipse and the green and blue segments are its axis. The resulting knot is presented on the right.*

As the ellipse is a parametric curve, we can compute the position of the current color in the 3D Cartesian space of Munsell COS by this simple equation:

$$\begin{aligned} colorPosition = & \; ellipseCenterPosition \\ & + \; (\vec{adir} * \cos(t * 2\pi)) \\ & + \; (\vec{bdir} * \sin(t * 2\pi)) \end{aligned}$$

with t the Hermite spline parameter, $0 \leq t \leq 1$,
$\vec{adir}$ the semi-major axis,
$\vec{bdir}$ the semi-minor axis.

In order to create a more harmonious knot, the t parameter can be adjusted so that, for example, the first chunk of a thread will go through half of the ellipse only while the second chunk will go through the other half. Indeed, it is desirable to highlight the symmetrical construction of the knot and repeat the pattern only when the general shape of the thread repeats. To clarify this, figure 10 shows an example.

Figure 10: *Illustration of the influence of the parameter t range. On the right knot, each chunk of each thread goes through the entire ellipse ($0 \leq t \leq 1$) while on the left knot, the first chunk goes through the first half of the ellipse ($0 \leq t \leq 0.5$) and the second chunk though the second half of the ellipse ($0.5 \leq t \leq 1$).*

Of course, the simplified version of the ellipse is a circle (eccentricity null), centered around the brightness axis. In that special case, the selected colors all have equal brightness and equal chroma. Only the hue varies in a uniform way through all the different hues present in the Munsell Book of Color. Figure 10 also illustrates this special case.

Finally, as in the case of *shadow* knots, it might happen that the number of colors an ellipse goes though is not sufficient for the colors to vary smoothly from one to another. In those cases, it is desirable to use color interpolation as detailed in the previous section.

5. Results and Discussion

To validate our results, we submitted a short and simple survey to 32 people through a web interface. The survey was not intended and did not serve as a reference to derive any algorithm presented in this paper. The goal was to validate our results by showing that our algorithms were providing rather aesthetic knots than non aesthetic ones. The observers were aged from 15 to 73 and came from different countries influenced by different cultures. Only 19% of them declared belonging to Celtic culture.

The first part of the survey intended to show that color and shape are the two main attributes that are taken into account when an observer selects the most aesthetic knot of a set. We presented them with 20 different knots, 12 of them were hand-made while the rest was computer generated and colorized with random saturated colors (we took the results from Kaplan and Cohen's paper [KC03]). 88% of the most aesthetic knots selected were hand-made ones, among which 71% were of type *rainbow*. The only *shadow* hand-made knot was selected 12% of the time, which is quite a high score considering that the probability of randomly selecting this knot was only 0.05. That's why we considered this type of knots also important. The remaining knots were of type *one color per thread* and the ones painted with just one color were more successful, which is quite logical as the ones colorized with several colors were the computer generated ones (i.e. non harmonious colors).

5.1. One Color per Thread

To see if knots generated with the first rule (same hue) are more successful than the ones generated using different hues as well as to see if the knots generated with 2 different hues with a ± 90 degrees angle are more successful than the ones using another hue angle, we proposed to the observers 7 series of knots, each of the series having a different number of samples but each sample representing a knot with the same shape and the same textured background (except for the first series as the purpose was to check the influence of the background). They were asked to choose the most aesthetic knot in each series, possibly providing no or several answers.

The survey showed that when people had to choose the most aesthetic knot between one where the two threads had the same hue and one where the two threads had a hue with a ± 90 degrees angle, the knot with the same hue was chosen only 56% of the time. At least, this proved the ± 90 degrees rule is not too bad. We could not exhibit a true preference for knots with a 270 degrees angle over the ones with a 90 degrees angle. We also included some knots using other angles: either very small (± 12 and ± 35 degrees) or of 180 degrees. The only emerging trend is that for dark knots, there was no real preference for any of the proposed angles (except very bad scores for very small angles of ± 12 degrees) while for brighter knots, the preference was for the ± 90 degrees of difference in hue angle 77% of the time. Figure 11 shows a few knots that participated in the survey and got a high score. We used a textured background to make it *old paper* like and put the knots in a more real situation. The drawback is that it might have influenced the color harmony. Outlines were added in some situations because above all, those algorithms are a tool for artists, easing the selection of colors, but still letting some room for personal improvement. In addition, in most Celtic designs, outlines are used.

However, as said earlier, ancient design was using rather dark and contrasted colors. Figure 12 shows a picture extracted from the Book of Kells and the knot we generated using the *equal-hue and equal-chroma* principle. Figure 13 shows the two possible colors to harmonize with a yellow: green or purple. This result is to be compared with figure 4 showing similar combinations.

Finally, we selected a few designs in a book [LeG99, LeG03] where knots have been colorized by artists by hand. As input to the system, we gave one of the colors present in the knot. We obtained the results of figure 14. The knots on the left are the original ones while the ones on the right are the ones generated with our system. The last series shows a design with 3 colors; left and right of it, the knots generated with our system. This last example could be a start towards three-colors harmony.

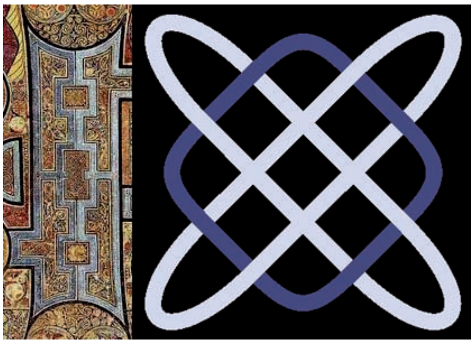

Figure 12: *On the right, a design extracted from the Book of Kells. Note the combination of the dark blue and light blue with the same hue and chroma. On the knot on the right, the input color was the dark blue. The light blue has been automatically chosen. Compare with the image on the left.*

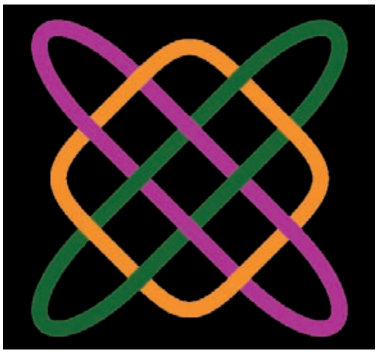

Figure 13: *This knot illustrates the two possible hue combinations with a yellow color on the same knot: green or purple. This image is to be compared with figure 4.*

5.2. Shadow & Rainbow

Following the algorithm detailed in section 4.3.1, we generated several knots with different hues. The results can be seen on figure 15.

Thanks to the algorithm presented in section 4.3.2 we randomly generated different *rainbow* knots by playing with the ellipse parameters. The results are shown on figure 16.

6. Conclusion and Future Work

We have proposed the first colorization technique for Celtic knots. The 2D nature of such knots makes it an excellent choice to apply color harmony principles. The reason for this is that Color Science studies are done by showing color samples side by side to observers.

Because our results are based on the Munsell COS, they are tied to the illuminant the color space was derived for. The second limitation of the algorithm comes from the mapping from the Munsell COS to the RGB space. Some colors are lost because they cannot be displayed on the screen. A way to overcome this problem would be to select Munsell color samples instead of RGB colors.

On the positive side, the results of this paper can be applied to a variety of other situations which are *display* based. Examples are the selection of harmonious colors to illustrate abstract concepts, selection of colors for web design, map coloring or any application involving 2*D* colorization problems.

As future work on Celtic knots, we would like to define a measure of the shape aesthetics and use this measure to automatically create harmonious shapes. As future work on Color Harmony, we would like to investigate three- and more colors harmony. As no work has been published regarding this matter in the Color Science field, we first need to set up a series of experiments in order to derive qualitative and possibly quantitative rules of harmony.

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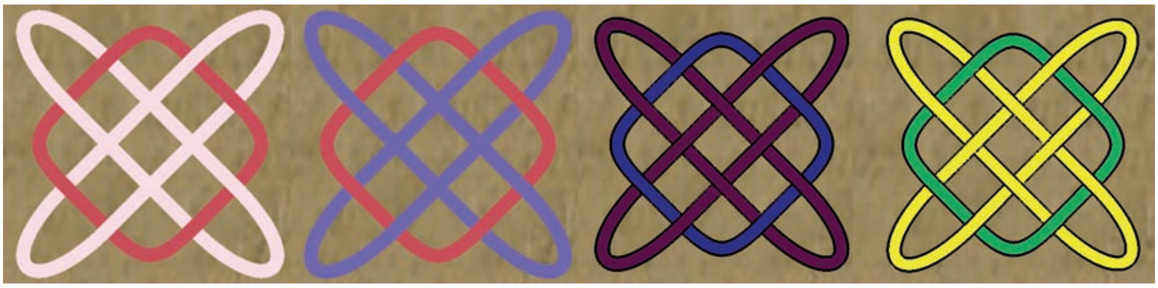

Figure 11: *From left to right: a knot with two colors of the same hue; a knot using the same first color, but a different hue at* 90 *degrees; an example of a knot with a hue angle of* 270 *degrees; another example of a knot with a hue angle of* 90 *degrees. All those knots obtained high scores in our validation survey.*

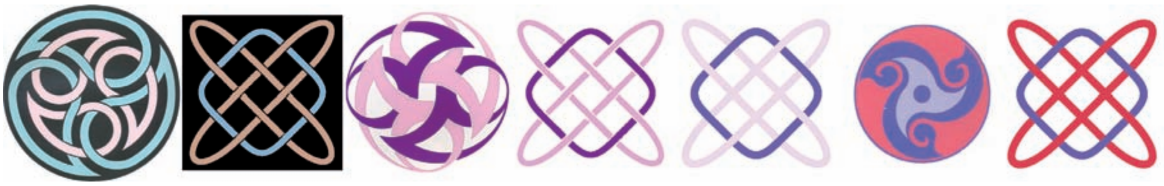

Figure 14: *Comparisons between hand-made knots and computer colorized ones. On the left of each pair, the original knot. On the right, the corresponding knot generated with one of the color as input. The last example uses* 3 *colors. Given the dark blue as input, our system proposed the two pairs of colors you can see aside the original design. Images extracted from [LeG99, LeG03]*

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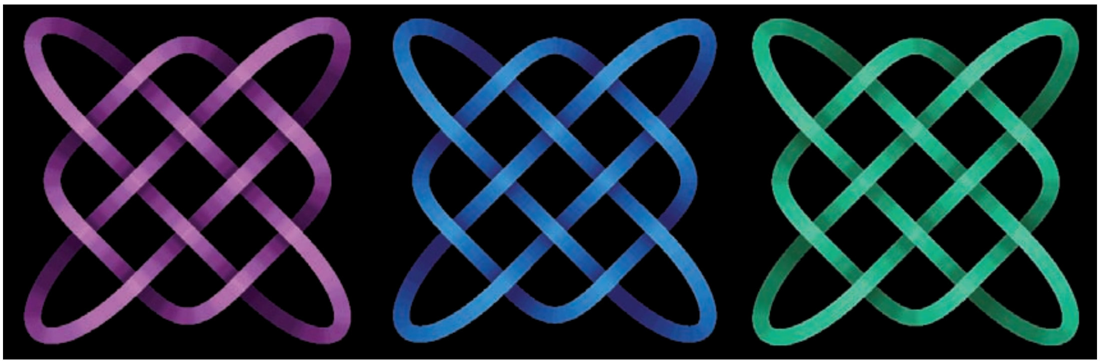

Figure 15: *A few* shadow *knots with different hues.*

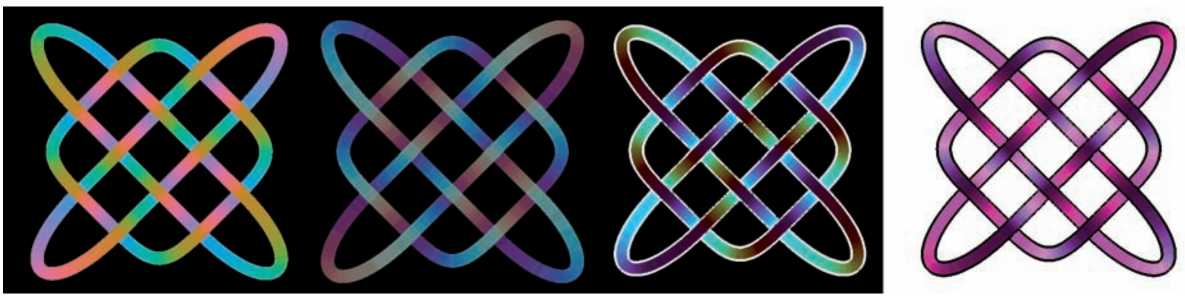

Figure 16: *A few* rainbow *knots with different hues and different ellipse parameters (position, orientation, size and eccentricity). We added outlines (black or white) to the two last examples to enhance the aesthetics of the knots by making them more conform to real Celtic knots. Note that we used black/white for the backgrounds and outlines so that it doesn't break the color harmony.*