Specular Manifold Bisection Sampling for Caustics Rendering

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Abstract

We propose Specular Manifold Bisection Sampling (SMBS), an improved version of Specular Manifold Sampling (SMS) [ZGJ20]. SMBS is inspired by the small and large mutations in Metropolis Light Transport (MLT) [VG97]. While the Jacobian Matrix of the original SMS method performs well in local convergence (the small mutation), it might fail to find a valid manifold path when the ray deviates too much from the light or bounces from a complex surface. Our proposed SMBS method adds a large mutation step to avoid such a problematic convergence to the local minimum. The results show SMBS can find valid manifold paths in fewer iterations and also find more valid manifold paths. In scenes with complex reflective or refractive surfaces, our method achieves nearly twice or more improvement when measured in manifold walk success rate (SR) and root mean square error (RMSE).

CCS Concepts

• Computing methodologies → Rendering;

1. Introduction

Path Tracing covers all paths in the Rendering Equation [Kaj86] theoretically, but not every path can be easily found in practice, such as the well-studied specular-diffuse-specular (SDS) paths. Recently Manifold Exploration [JM12] and Manifold Next-Event Es-
2. Prior Work

Light Transport Simulation has been an important topic in computer graphics for many years. The original Ray Tracing [Whi80] simulates paths with reflection and refraction. The advent of the Rendering Equation [Kaj86] led to the popularity of Path Tracing, a Monte Carlo integration technique that simulates all possible light paths through random walk.

While Path Tracing simulates all possible paths to get a realistic picture, the random walk from the eye could miss the light to form a valid path, leading to long convergence time to form a noise-free picture. In contrast, Direct Lighting [SWZ96] and Bi-Directional Path Tracing (BDPT) [LW93] can efficiently find a valid path by forcibly connecting to the light.

BDPT generates eye paths and light paths by random walk before connecting to the light points, so it is still difficult to find the paths that pass through a small gap, such as the scenes where the light is behind an ajar door. In addition, the specular-diffuse-spectral path is also difficult to be found through forcible connection because of the material constraint. Metropolis Light Transport (MLT) [VG97][KSK01] first finds enough valid paths through BDPT, and then reuses these paths by mutation (disturbance) to quickly find many different valid paths.

Our work focuses on caustics rendering within the path tracing framework. The classic Photon Mapping [Jen96] offers a fast yet biased rendering method for caustics. The Vertex Connection and Merging (VCM) [GKDS12] offers a caustics rendering method by combining the Bidirectional Path Tracing and Photon Mapping with multiple importance sampling. Recently Manifold Exploration [JM12], Manifold Next Event Estimation (MNEE) [HDF15], and Specular Manifold Sampling (SMS) [ZGJ20] were proposed to quickly find the valid caustics paths. We will discuss them in more detail in Section 3.

3. Background

The caustics effects in the scene are caused by refractive or reflective surfaces. Take Figure 2 as an example. The caustics appear on $x_1$ when the light from $x_4'$ is refracted by the transparent sphere. However when we perform path tracing, the initial path starting from $x_1$ might find $x_2, x_3, x_4$ and miss the light at $x_4'$. Manifold Exploration [JM12] offers a solution. The first step generates $x_1, ..., x_4$ by random walk. Although it has not found the light at $x_4'$ initially, the main purpose of this step is to get all the manifold points of reflection and refraction on the path. The second step uses the curvature constraint of each point (e.g., $c_2$ and $c_3$) to build a constraint matrix $C$, which then yields the Jacobian matrix $\nabla C$. This allows us to move toward the light $x_4'$ by $A^{-1}B_4\Delta x_4$ to find the desired $x_2$. Repeat steps one and two until the ray hits the light. Algorithm 1 shows the Newton down-hill method to speed up the approximation, as outlined in [JM12]. After finding the path, the unbiased result is obtained by the generalized geometry factor in Figure 3.

Manifold Next Event Estimation (MNEE) [HDF15] applies Manifold Exploration to Next Event Estimation (NEE) to connect the path to the light even with the presence of refraction. It is in particular effective to solve the difficult specular-diffuse-specular (SDS) cases. In addition, MNEE improves the equations of Manifold Exploration in Figure 2 by changing the step of finding $\Delta x_2$ from $\Delta x_3$ to finding $\Delta x_2$ from $\delta c_3$.

MNEE can find the caustics caused by refraction but not the caustics caused by reflection. Specular Manifold Sampling (SMS) [ZGJ20] samples all specular objects including reflective surfaces to address the issues with the reflected caustics path. SMS also improves MNEE further by changing the half-vector based constraint function of MNEE to angle-difference based constraint function. The improvement can be clearly seen from Figure 4. In addition
4. Specular Manifold Bisection Sampling

We propose Specular Manifold Bisection Sampling (SMBS), an improved version of SMS [ZGJ20]. SMBS is inspired by the small and large mutations in Metropolis Light Transport (MLT) [VG97]. While the Jacobian Matrix of the original SMS method performs well in local convergence (the small mutation), it might fail to find a valid manifold path when the ray deviates too much from the light or bounces from a complex surface. Our proposed SMBS method adds a large mutation step to avoid such a problematic convergence to the local minimum. Figure 5 explains the difference between the small mutation and the large mutation. When any of the specific conditions in Figure 6 occurs, we trigger the large mutation step. Otherwise the small mutation step of the original SMS is followed. The purpose of the large mutation is to find a better search area, which is then followed by the small mutation to approach the correct path. The following subsections further explain our SMBS method in detail.

4.1. Generating the Initial Path

Regardless of MNEE, SMS, or our SMBS, the initial path must be generated, and then the answer can be slowly approached based on this path. First, trace a ray which starts from the eye. Second, randomly sample a specular surface and a light surface when the ray hits a diffuse surface by random walk. Taking the diffuse surface at $x_1$ as an example, shoot a ray from $x_1$ to the specular surface $x_2$ and get the initial path $x_1, x_2, \ldots, x_n$ through multiple refractions and reflections ($x_n$ is non-specular). If the light surface $x_n'$ is not hit in the end, it will use mutation to find the path from $x_1$ to $x_n'$. See 4.2 for a detailed mutation process.

4.2. Large Mutation

The path mutation method of SMBS is shown in Algorithm 2. Compared to the previous Algorithm 1, our method adds a condition of large mutation in line 3. We perform a large mutation when the path $x_1, \ldots, x_n$ satisfies one of the conditions described in Figure 6, otherwise we use the small mutation of the original SMS. The first condition occurs at the total internal reflections during a refraction such as the cases in Figure 1. The second condition avoids the back-lit situations where the small mutations usually get stuck. Figure 10 shows an example of such cases. The third condition is designed to avoid too many small mutations which could be costly. Figure 12 offers an example of such cases. Our experiments show
of the manifold walk success rate (SR), which means it is easier to find manifold paths. In addition, a good mutation also increases the sampling direction has lower average mutation numbers per path (mpp), which means that a manifold path can be found through fewer mutations. In addition, a good mutation also increases the manifold walk success rate (SR), which means it is easier to find the manifold paths.

### 4.4. Unbiased Results

Finally, we use the same Algorithm 5 as the SMS to get unbiased results. First find a manifold path. Then get the PDF, \( p_b \) in Algorithm 5, of the path by randomly sampling manifold paths.

### 5. Results

We modify the open-source version of SMS which is based on Mitsuba2 [Jak20]. Regarding the program, we mainly modified the
Algorithm 4: GetRatio

Input: The first vertex \( x_1 \) and the second vertex \( x_2 \) in the path. The previous direction \( v_1 \) and the current evaluation direction \( v_2' \).

Output: A interpolation coefficient \( \alpha \).

1. \( \alpha = 0.5 \)
2. If \( \text{dot}(v_1, -v_2') < 0 \) then
3. //backlight such as the right of Figure 5
4. \( \alpha = 1 \)
5. end
6. While true do
7. \( \text{dir} = \alpha(-v_2') + (1 - \alpha)v_1 \)
8. If boundingBox \( (x_2) \).rayIntersect \( (\text{dir}) \).isValid() then
9. break
10. end
11. \( \alpha = \alpha + 0.5 \)
12. end
13. Return \( \alpha \)

Algorithm 5: UnbiasedSMBS

Input: The shading point \( x_1 \) and the light point \( x_0 \) with density \( p(x_0) \).

Output: Estimate of radiance traveling from \( x_0 \) to \( x_1 \).

1. \( x_2' \leftarrow \) sample a specular vertex
2. \( x_2^2 \leftarrow \) the second vertex returned by OurWalkManifold (alg2)
3. \( (1/p_k) = 1 \)
4. While true do
5. \( x_2' \leftarrow \) sample a specular vertex
6. \( x_2^2 \leftarrow \) the second vertex returned by OurWalkManifold
7. If \( ||x_2^2 - x_2'^1|| \leq \epsilon \) then
8. break
9. end
10. \( (1/p_k) = (1/p_k) + 1 \)
11. end
12. Return \( f \langle x_2^2 \rangle G(x_1 + \ldots + x_n) (1/p_k) L_e \langle x_n \rangle / p(x_n) \)

Figure 8: Two scenes showing specular reflective and refractive surfaces, rendered with unbiased SMBS. We show equal-sample comparison (1 spp) between using \(-v_2'\) in Figure 7 as the next sampling direction (left) and using interpolation coefficient to get the next sampling direction (right). Insets show average mutation numbers per path (mpp), manifold walk success rate (SR), and root mean square error (RMSE).

Figure 11 shows that under the equal time, SMS can achieve 2 spp, while SMBS only achieves 1 spp. We find that under the same spp, SMS only need to process about half of the number of rays in SMBS, leading to faster rendering. However the values of success rate show that the number of paths successfully found by SMBS in this scene is about 9 times higher than that of SMS. Therefore, even if SMS has a higher spp in this scene under equal time, our method still has lower RMSE and produces better results.

In the normal-mapped reflection scene of Figure 12, SMBS not only spends less time, but also has lower RMSE. The number of rays of SMBS is about half of SMS because SMBS has lower number of mutations per path (mpp). A lower mpp means that SMBS can spend fewer mutations to find a correct manifold path. The success rate of SMBS is also improved because large mutations reduce the probability of local convergence.

We also compare our results to the two-stage SMS [ZGJ20] in Figure 12. As mentioned previously in 4.2, the stage one of the two-stage SMS may be considered a certain kind of large mutation. However, its large mutation is performed once at the initial stage and already has a very high manifold walk success rate, our SMBS can further improve the results because of our large mutation conditions (Figure 6) that detect the back-lit cases.
while our SMBS method may perform large mutation whenever it is necessary. Furthermore, while the two-stage SMS works mainly on normal-mapped reflection, our large mutation works on all types of specular surfaces.

6. Conclusions and Future Work

Our SMBS method avoids the problematic convergence to the local minimum in SMS. In scenes with complex reflective or refractive surfaces, our method achieves nearly twice or more improvement when measured in manifold walk success rate (SR) and root mean square error (RMSE).

We have not yet implemented our SMBS method on GPU. How to convert the current method into an algorithm that is more conducive to parallel processing is a possible direction for future work.

The higher manifold walk success rate (SR), the more manifold paths found under the same spp, and the less noise. Therefore, increasing SR helps the convergence of a picture. We think that SR may be further improved by controlling the specular sampling. For example, we may reduce the sampling probability of back-lit specular surfaces because they are less likely to fall on caustics paths.

7. Acknowledgment

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References


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Figure 9: Results of the refraction scenes in roughly equal time (~14 seconds). Insets show samples per pixel (spp), number of rays, manifold walk success rate (SR), and root mean square error (RMSE).

Figure 10: Results of the scenes with backlit area in roughly equal time (~5 seconds). Insets show samples per pixel (spp), number of rays, manifold walk success rate (SR), and root mean square error (RMSE).
Figure 11: Results of the refraction scenes in roughly equal time (~15 seconds). Insets show samples per pixel (spp), number of rays, manifold walk success rate (SR), and root mean square error (RMSE).

Figure 12: Results of the normal-mapped reflection scene in 1 spp. Insets show running time, average mutation numbers per path (mpp), number of rays, manifold walk success rate (SR), and root mean square error (RMSE).