The 11th International Symposium on Virtual Reality, Archaeology and Cultural Heritage VAST (2010)
A. Artusi, M. Joly-Parvex, G. Lucet, A. Ribes, and D. Pitzalis (Editors)

Natural Lighting, Gilts and Polychromy of *Notre-Dame de Paris* Cathedral

P. Callet[1,2,3] and S. Dumazet[3] and C. Leclercq[4,5,6] and C. Politi[5]

[1]Gunzo project, Arts & Métiers ParisTech, Cluny, France
[2]CFC, Centre Français de la Couleur, France
[3]MAS Lab. Ecole Centrale Paris, France
[4]LETA-LAM, Université Paris I Panthéon-Sorbonne, France
[5]Ecole du Louvre, Paris, France
[6]CITU, Université Paris 8, France

Abstract

*Lighting either natural or not in the european medieval architecture (roman period) is studied and simulated around a special and famous example. "*Notre-Dame de Paris*" cathedral and its polychromy are studied for many years and, today the remains of polychromy are very reduced. This prestigious monument is then a unique opportunity to study the complex relationships between light, paint and gilt. Monumental polychromy of the religious buidings is now well established and some attempts exist for retrieving a kind of optical effects well controlled in the medieval era. An interdisciplinary study which derived from the french project TerraNumerica allowed to value a historical hypothesis about the possible optical role of gilts. Thanks to the simulation of a global illumination in spectral rendering operating with the photon mapping algorithm, a new interpretation of polychromy effects is proposed, including the lightguide effect produced by the gilts. Many measurements on formulated polychrome samples and spectrophotometric captures concerning natural lighting at sunset were made to validate that new hypothesis. The obtained results guided the interdisciplinary team on a new focus on the polychromy restitution of the medieval buildings and monuments. It became quite evident that the medieval sculptor brought a very high and accurate mastership in lighting. Playing with the symbolic and colored light for illuminating the* Galerie des Rois *(the kings of Judah and not the kings of France) he offered to the christian people a very "special effect" for the last minutes of the day time. Physically based simulations presented here show a brand new hypothesis concerning the optical role of gilts.*

1. Introduction

The french TerraNumerica project permitted to obtain many 3D data on the historical centre of Paris. Within the scientific and technological framework offered by such an interdisciplinary project, a small part was dedicated to the study and restitution of polychromy. Many previous works were driven on such a subject with museums (Musée national des arts asiatiques-Guimet, basilique de Saint-Denis, Louvre, Bibliothèque Nationale de France,...) giving a good experience for computer scientists in collaborating with physicists, chemists, engineers, art historians, archæologists and curators. Several specialized laboratories and companies worked together in the project TerraNumerica. We focus our study on the main and original results regarding a famous monument. Previous scientific studies emphasized hypotheses concerning the possibility of the optical role of gilts. Thanks to the simulation of a global illumination in spectral rendering operating with the photon mapping algorithm, a new interpretation of polychromy effects was proposed, including the lightguide effect produced by the gilts. Several measurements concerning natural lighting at sunset were made to validate the new hypothesis. The obtained results guided the interdisciplinary team in a special way where a new focus on the polychromy restitution of the medieval buildings and monuments were very intellectually stimulating. It became quite evident that the medieval sculptor brought a very high and accurate mastership in lighting. Playing with the sym-

DOI: 10.2312/VAST/VAST10/063-070

bolic and colored light for illuminating the *Galerie des Rois* he offered to the christian people a very "special effect" for the last minutes of the day time. Our work mainly concern the set of sculptures shown in Figure 1.

2. Historical context

TerraNumerica is a french project lead by Thalès wich involves many French laboratories (ECP, ENSMP, IGN, etc.), universities (Université Paris 1, Université Paris 8, etc.) and enterprises (MENSI-TRIMBLE, etc.). It aims to develop and integrate technologies for automated 3D modeling of urban environments. This technological research project has been conceived first as platform of production able to improve the productivity and the realism of 3D urban modeling, and, secondly, to visualize it on several platforms: Web services, Mobile application, Virtual Reality (a CAVE-like environment) and Augmented reality displays. The project was divided in several work-packages (production, reconstruction, etc.), including one dealing with user stories or scenarios. Indeed, one of the most interesting part of this project was to bring together researchers from different disciplines, e.g. sciences as well as humanities. Brainstormings on different scenarizations in urbanism, security services, art and games took place at the begining of the project, keeping always in mind to highlight the different partners works and to deal simultaneously with heritage, digital technologies and a scientifical research. Several data acquisition technologies were involved in this project. The main efforts consisted in reconstructing a more or less repetitive architecture such as the haussmanian style architecture. However, as it was not possible to automatically reconstruct very specific monuments, it was proposed to scan with MENSI-TRIMBLE 3D laser technologies a selection of three monuments. We decided to scan the "Musée d'Orsay", the "Place du Châtelet" and the "*Notre-Dame de Paris*" cathedral. This last exemple was an opportunity to involve the Laboratoire Mathématiques Appliquées aux Systèmes (ECP) along with MENSI-TRIMBLE team (and costly technologies a museum or a monument could not afford because it is not considered as a priority), and a medievalist teacher working at Ecole du Louvre. Thus the interdisplinary study permitted to enhance an historical hypothesis about the possible optical role of gilts. That yellowing of the natural light at the beginning of the sunset quickly turning orange-red in the end, created a kind of aureola behind the heads of the kings' sculptures. Regarding optics that special lighting could have been produced by a caustic effect focusing the sun light on the naked stone and, thus, back-lighting the heads in a multiple scattering process. The complete scientific way for sustaining our hypothesis is successively described. Firstly, we expose what are the documents and observations known on *Notre-Dame de Paris* and its polychromy. Secondly, the 3D acquisition process, the amount and treatments of collected data are presented. Thirdly, the measurements made on materials and light are briefly indicated for introducing them in the spectral simulation. Physically based simulations presented here show a brand new hypothesis concerning the optical role of gilts in the medieval era. During a cloudy day or under a very directional solar light in the afternoon, the upper arcatures just above each statue of king are always in the shadow. For a very intense direct lighting, the cast shadows are very important. For dissimulating these hard cast shadows the ancient greeks painted the backward wall of a pediment with a dark blue colour.

Figure 1: *During a cloudy day, a general view of the median part of the* Galerie des Rois *at* Notre-Dame de Paris *cathedral. The sculptures were handcrafted during the XIXth century under the restoration campaign led by the famous architect Eugène Viollet le Duc.*

Figure 2: *Original remains of the heads of the kings of Judah at Musée National du Moyen-Age in Paris [EB82].*

3. 3D acquisition

As already mentioned above, the 3D data required were captured thanks to the knowledge and technique of the Trimble-

MENSI company. Three prestigious monuments were se-

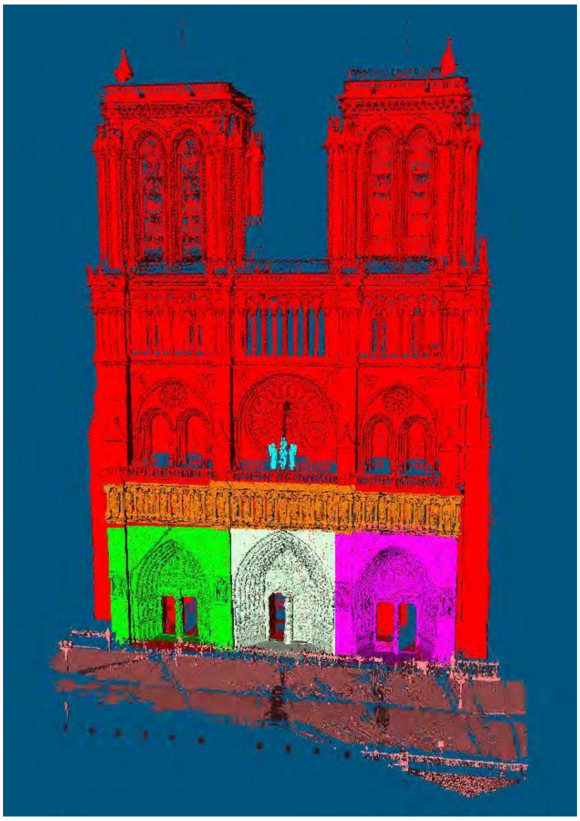

Figure 3: *Results of the 3D scanning process obtained by the 2 GX and FX set of sensors. About 53 millions of 3D points were acquired.*

lected for 3D digitization in the centre of Paris. The Notre-Dame de Paris cathedral is probably the most famous of them. The scanning process was made in the following conditions:

- 5 places were choosen for a GX device and 3 among these were accurately resolved by the Traverse method. The 2 others were resolved using the "Cloud Based Registration" software (RealWorks).
- 3 places were choosen for a FX device and used for the 3 great doors ; the captured data were resolved and assembled using spheres then the global Cloud Based Registration software (RealWorks).
  The complete 3D points cloud of 52 972 755 elements was divided into 3 subsets: facade, pertinent details (Kings gallery, the 3 great doors, the Virgin Railings) and the public road. The pertinent details subset exist in two levels of detail at 10 and 5 mm of spatial resolution. The rest of the monument is acquired at 50 and even 250 mm resolution.

4. Remains of polychromy

In the middle age, the symbolic of colours, their associations and even the order of interpretation (from back to front in the painted image) are very different from what they are today for us [Pas90]. Lighting conditions, inside or outside are completely different too. The spectrum of the white or natural light is not known and the colours classification does not match at all our occidental admitted repartition of today. Colours as they are more symbols than materials could not be disposed at random nor mixed together ; thus, it is very surprising to find a mixture of pigments on a middle age sculpture [EB75, EB93]. The presented work is mainly founded on the results obtained ten years ago and also more recently that were mentioned in the french meeting hold in Amiens [Fon00]. The discovery of some traces of a reddish substance on the inner surface (intrados) as shown in Figure 4 was then the starting step of the questioning on a special optical role of gold.

4.1. Light effects

Colour was much more defined by a certain sensibility to light in the Middle Ages, even in the texts and sometimes translators used modern word to translate what was more in Latin an impression of light than a "real" colour (colour surface) [Pas90]. This idea of light was also emphasized by contemporaneous writers, like Hugues de Saint-Victor (1096-1141) in his *De tribus diebus:* "What is more beautiful than light which is without colour but colours everything by lighting?" [dSv55]. And there was also an allusion to "God Who is Light". Thus, remains of polychromy have to be reinterpreted thanks to iconography and effects of sunlight on the fronts of cathedrals. Viollet-le-Duc noticed that the facade of *Notre-Dame de Paris* was covered by lots of painted and gilded parts such as the *Galerie des Rois* all coloured and gilded. A few sentences later, he explains a medieval technique consisting in placing glass sheets covered with gold or tin sheets in the back of the painted sculpture [VIO56].

4.2. Iconography and Polychromy

The facade of the cathedral depicts an iconographical program based on the cult and glorification of the Virgin Mary. Originally, the first sanctuary was dedicated to St. Stephen, but the development of the cult of the Virgin in the beginning of the 13th century and the link made between Mary and the Church explain why the cathedral became "*Notre-Dame de Paris*". Two of the three portals depict stories of the Virgin's mother, St. Anne, and the other scenes from life of Virgin Mary with the Coronation and the Dormition. The third portal is about the Last Judgement, theme which is classical in medieval iconography, and particularly on cathedral's portals. Apparition of the *Galerie des Rois* on *Notre-Dame de Paris* must be related to the cult of the Virgin. It actually represents the kings of Judah, from Jessé to Joseph, according to St. Matthew's Gospel (1, 6-16). The 28 original kings were represented under arcatures but soon a contemporaneous text named *Les XXIII manières de vilains* spread the idea the sculptures were the representations of kings of France. Thus when the French Revolution broke out, the kings were thrown down on the parvis and soon beheaded. Nowadays, we can only see on the facade 19th century copies made for the restoration supervised by Viollet-le-Duc, but in 1977, fragments of 21 heads were found and are now preserved in the *Musée national du Moyen Age-Thermes de Cluny* in Paris. Polychromy was found in the remaining heads and also on some details of architecture, as the 1996-2000 restoration proved it [Fon00]. Almost all of the kings were bearded man, wearing full coat open on a tight dress around the waist, crowned and holding a sceptre in the hand. Hair and beard were painted in yellow, blue-grey or red colours, faces were painted in order to copy the colour of flesh with a few pink highlights on the cheeks, on the nose and on the ears. Eyes were surrounded with a black line and eyelids sometimes shadowed by a red line [EB82]. Polychrome Kings of Judah were certainly partly gilt, like the statues and representations of the portals. But unusual located polychromy, for example in the back of the trefoil arcatures of the *Galerie des Rois*, can be easily explained by the light projected by the sun, and particularly at sunset. Sunbeams projected on the facade played with polychromy, emphasizing the iconographical program. When portals were already in darkness, the *Galerie des Rois* remained colourful, enlightened and gilt, as the kings, Christ's ancestors, were protecting Christians. Polychromy in the medieval era was very extended on the monuments. It is well established that all sculpted parts and all ornaments were painted and sometimes gilded. Here the optical role of gold is not only metaphoric because the noble metal is always considered as "light", "material" and moreover "symbol". Light is always associated to God so that cast shadows are produced by the material world expressing a complete opposition to the immaterial one. Very well exposed to the assaults of weather, the western facade had many difficulties to preserve its polychrome attributes through centuries. Internal remains of painted parts are, for that meteorological reason, more frequent independently of any further restoration. Notice that, according to the real process of painting on stone, the coloured effects obtained by simulation are quite different from a polychrome restitution by a direct electric-lighting on the real historic building (and paradoxically, during the night).

5. Optical simulation and modeling

Art materials and more precisely painting materials ar studied for a long time and their properties are mainly referenced in [GET, Bri80]. In computer graphics, the simulation of pigmented media such as paints and plastics is not very new [HM92, Cal96] but always useful and in progress thanks to spectrophotometry using portable devices [CS01, Cal05].

Figure 4: *The remains of a mixture of reddish pigments made of ochre and read lead over a white lead (ceruse) preparation layer. Image extracted from [Fon00].*

Painting materials during the middle age are not very different from those known in Antiquity. The classical organization of the paint layers deposited for polychromy are, from the innermost to the outermost: stone, white lead (ceruse = lead carbonate), first pigmented layer, last pigmented layer. The first paint layer is not always present but, generally it fulfils an economic function. For painting a king statue the most expensive and precious pigments were used by the artist and disposed in the thinnest external layer. However to reinforce the optical effect of those noble pigments the artist did not cover the white lead preparation layer with the most expensive material first. For example, a painting with natural ultramarine blue (the most precious pigment) will be optically more efficient if deposited over a first layer already blue (e.g. an azurite layer). Some recent works had given us experience in modeling the influence of the bole colour on the visual appearance of gilded statues [DAGdC*07]. The model is described in Figure 7 where the inner coating was ceruse and the bole layer made of a mixture of 50% red lead and 50% red ochre on which were deposited the gold leaf fixed by an adhesive substance called mixtion. That last metallic layer was described from the knowledge that we

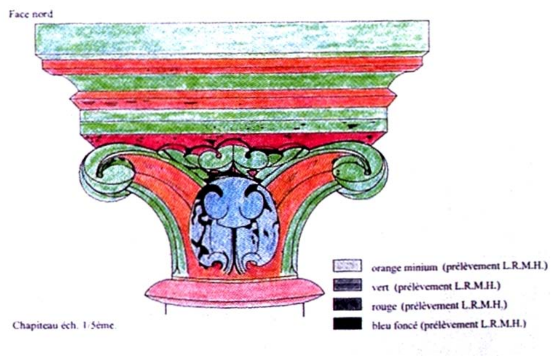

Figure 5: *The retrieved polychromy composition and disposal of a typical chapiter situated on the occidental facade. Figure extracted from [Fon00].*

have about the medieval techniques of gilding. Notice that for obtaining a good shining effect with a golden surface, convex shapes are required and disposed in such a way that the solar light strikes these shapes and plays with multiple inter-reflections. Here, at the contrary, the golden parts are concave and oriented for receiving the light only at sunset. Thus a moderate porosity and cracks distribution inside the gold leaf itself was incorporated in the simulations. The influence of the colored preparation (bole) on the visual appearance of the metallic gold leaf is modeled according to the now well known Kubelka-Munk theory of pigments mixing and layered scattering materials (paints !). The multiple and internal scattering of light inside the bole layer gives a slightly reddish tone to the gilt. The metallic influence is rendered thanks to a model including a measured roughness but incorporating at each step of the computations the polarization state of light. The effective complex index of refraction of a thin metallic film such as a gold leaf is not known. To avoid this problem of missing data we used the optical constants of the bulk metal modified by a Maxwell-Garnett theoretical approach and depending on the surfacic density of cracks and holes. The model is shortly described hereafter. The Max Born notation for the index of refraction is used throughout the model, thus :

$$\hat{n}(\lambda) = n(\lambda)[1 + i\kappa(\lambda)] \quad (1)$$

where $n(\lambda)$ is the real part of the index of refraction and $n(\lambda)\kappa(\lambda)$ its imaginary part. The corresponding data used for spectral rendering are given in Figure 6. Virtuelium software uses a layer description of many classes of materials. The structuration of a material is then read in the 3D scene file written in XML format with all other useful indications for rendering (CIE reference observer, number of wavelengths to account for sampling, illuminants, 3D meshes, etc.)

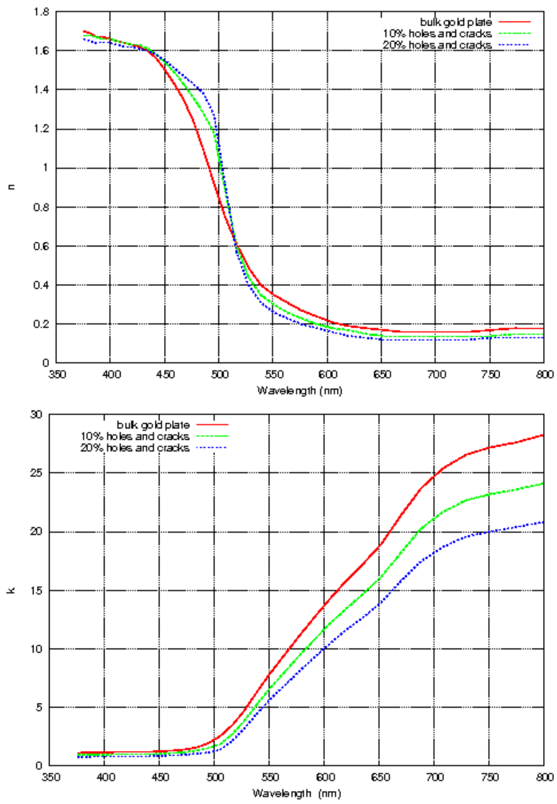

Figure 6: *The complex index of refraction of bulk gold used instead of the effective ones, unknown for thin metallic films. An approximation is made to compute a plausible index of refraction according to the Maxwell-Garnett theory of effective media. The set of curves in the top figure give the variations of the real part of the index of refraction while the other indicate the imaginary part.*

5.1. Natural lighting

Sunset was at 21:49 on 2009 july 15th so that we get many emission spectra thanks to our MAYA 2000 spectrophotometer. The experimental setup is quite simple and gives very detailed information on the yellowing and amplitude of the solar lighting. For normalization a first measurement is made at 12 UT at a fixed angle between the collecting optical fiber and the reference white oriented in the direction of the solar disc center. All the measurements were obtained in similar conditions with the same geometrical observation parameters. Thus, the recorded spectrophotometric measurements are given in Figure 8. Our MAYA device has a very high sensitivity ranging in 200-1100nm in wavelength with a very high resolution ($\leq 0.5nm$). The formulated model is derived from the classical Cook-Torrance model with roughness parameters. The proposed algorithm is briefly described here-

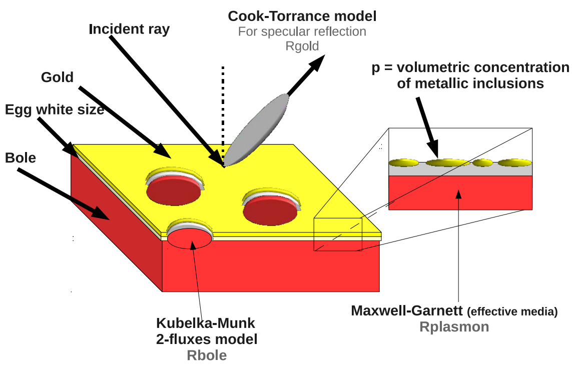

Figure 7: *The gilded surface model used throughout the simulation.*

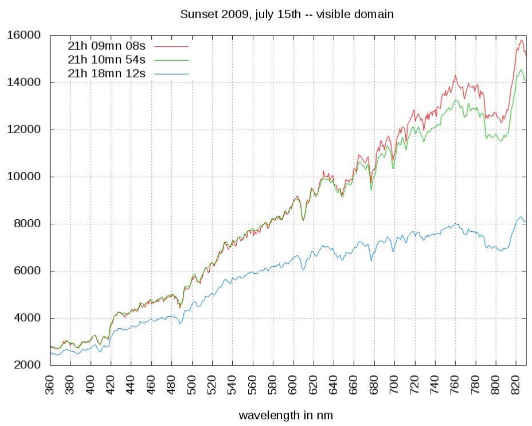

Figure 8: *Rapid evolution of the solar spectrum at the beginning of sunset on 2009 july 15th at 21:09, 21:11 and 21:18 from top to bottom.*

after. For the electronic plasma contribution (surface plasmons) modifiying the spectral transmissivity of the gold leaf, we have calculated an effective index of refraction of the gold leaf [Abe63] according to a model of effective medium derived from Maxwell Garnett theory [Gar04]. Let us consider a set of metallic inclusions of volumetric concentration p embedded in a homogeneous dielectric medium as described in Figure 7. We successively calculate the effective electric field acting on such a metallic inclusion. In reaction to this excitation the particle modifies the applied electric field and the electric displacement too. This mechanism leads to an effective dielectric function ε_e depending on the geometrical form of the metallic inclusions. The optical constants of bulk gold are available in the scientific literature but our optical data for bulk gold have been obtained by spectroscopic ellipsometry. Let us also consider, for generality, some lenticular particles having their short ellipsoidal axis c perpendicular to the horizontal delimiting faces of the global medium ; the two other axes are such $a = b$ parallel to the horizontal interfaces. Viewed from above such a system could be described as a "go ban" with a unique kind of stones and where the go ban itself is the hosting dielectric medium. We define the excentricity as

$$e^2 = \frac{a^2}{c^2} - 1 \qquad (2)$$

and the depolarization factor

$$A = \frac{1+e^2}{e^3}(e - \tan^{-1} e) \qquad (3)$$

Being given the two complex dielectric functions of gold and that (approximated) of the hosting medium, i.e. the egg white size (*clara ovorum*), we have, the following *complex effective dielectric function*:

$$\varepsilon_e = \varepsilon_m \frac{\varepsilon_i[A + p(1-A)] + \varepsilon_m(1-p)(1-A)}{\varepsilon_i A(1-p) + \varepsilon_m[1 - A(1-p)]} \qquad (4)$$

All the ε depending on wavelength, we used the definition of the *complex effective index of refraction* $\hat{n}_e(\lambda)$ such that :

$$\hat{n}_e(\lambda) = \sqrt{\varepsilon_e(\lambda)\mu_e(\lambda)} \qquad (5)$$

In this last eq. (5), the magnetic permeability $\mu_e(\lambda)$ was considered equal to unity as it is easy to verify in the visible domain. The reflectivity and transmissivity functions governed by the Fresnel formulas mainly depends on the imaginary part of the index of refraction defined in eq. (1). That effective index of refraction [CCCA73] has a lower imaginary part than pure gold and, consequently its spectral transmissivity is magnified. In this way the fraction of light diffusely emerging through the metallic leaf and originating in multiple scattering inside the volume of the bole layer is accessible. The effective medium formed by the metallic film, the nanoscopic holes and the egg white size is approximated by a Maxwell Garnett model of the dielectric function. In eq. (4) the optical absorption depends on the volumetric concentration p. R, the reflectance factor and p vary in opposite way. The upper value of p is fixed while no percolation process appears inside the effective medium made of holes and gold platelets. ε_m is simply computed with an effective index of refraction for egg white size, i.e. $n_{egg} = 1.36$. Using the diffuse reflectance factor spectrum of the bole, considered as a completely opaque and multiple scattering pigmented medium, we add this contribution to the global reflectance factor. The main fraction of the reflected light is governed by Fresnel laws modulated in amplitude by an optical roughness. This term is the only classical Cook-Torrance one. The roughness influence is extracted from laser profilometric measurements. The diffuse reflectance factor emerging through the metallic leaf and from the bole is obtained by the Kubelka-Munk model for opaque layers whose parameters (the K/S factors) are inverted from the diffuse reflectance factor R_∞ of the only bole.

$$\frac{K}{S} = \frac{(1-R_\infty)^2}{2R_\infty} \quad (6)$$

The ratio defined in eq. (6) is important for adjusting the mixing of pigments while it is possible to use R_∞ directly for tests.

5.2. Proposed Algorithm

The previously mechanisms of light-matter interaction are then abridged in 3 components. In this set of interactions, the two macroscopic components are the most classical. The summation of the three independent mechanisms gives the received luminance. Thus, the regular reflection on the moderately rough gold surface, R_{gold} and the diffuse reflection from the missing parts from the bole R_{bole} is added to the plasmon surface effect $R_{plasmon}$. The Cook-Torrance model modulated by the optical roughness of the metallic film is used to compute R_{gold} thanks to the complex index of refraction acquired by spectroscopic ellipsometry. The Kubelka-Munk model is used for R_{bole}. The last term $R_{plasmon}$ is more complicated. The effective interaction area is used for weighting these three terms.

$$R = fR_{bole} + (1-f)((R_{gold} - T_{pgold}) + R_{plasmon}) \quad (7)$$

where f describes the ratio between the cumulated hole area and the global gold leaf area. T_{pgold} defines the gold transmissivity due to surface plasmon resonance. We thus consider an effective dielectric-metallic medium made of a metallic continuum embedding the holes in which an also effective resin is present and composed by the egg white size and air. The model described here leads to the computation of an effective index of refraction for the surface plasmon component depending on the holes and cracks concentration. Two estimations for 10 % and 20% are presented in Figure 6.

5.3. Rendering with Virtuelium

Spectral rendering has now a relatively long history in the computer graphics community [Hal89]. The radiative transfer equation was algorithmically solved for diffuse materials with the radiosity rendering softwares [NCNS03]. Another category of global illumination algorithms was proposed by Jensen at the end of the 90s. For combining the advantages of a spectral ray-tracing rendering and radiosity-like light transport effects, we used the Virtuelium open-source software. The last version of the Virtuelium open-source software was used for simulating the natural ligthing on *Notre-Dame de Paris* facade at sunset. Virtuelium is implemented for accounting for the most intrinsic characteristics of materials (complex indices of refraction) and necessarily extrinsic parameters describing how are the materials used in the 3D scene (powder, bulk, mixture, solution, composite,...). The computed images based on the optical properties measured on all the materials of the sculptures are computed with a grid and cluster facilities. The simulation conditions for all the computations include a colorimetric observer CIE 1964 (10°). Some of the main characteristics of Virtuelium are:

1. Spectral rendering with a parametrable wavelength sampling for all element (viewer, materials, illuminants, propagation media) ;
2. Polarization effects accounted for at all step of the computations ;
3. XML formalism for the 3D scenes and data descriptions ;
4. Multithreading and/or grid computing ;
5. Multilayered materials (macroscopic layers of paints or multiple thin films assembly) ;
6. High Dynamic Range Imaging ;

A special photon-mapping algorithm according to Jensen [Jen97, Jen99] specifications is implemented in Virtuelium software and used here for realistic rendering (see http://virtuelium.free.fr).

6. Conclusions

There are still many things to do for achieving and, why not, for viewing a complete proposition of polychromy restitution of *Notre-Dame de Paris* cathedral. A spectral simulation with Virtuelium software offers many capabilities that were very different from how we perceive them today about the polychrome appearance of many historic buildings. We have shown that a scientific visualization founded on spectral simulation of the interaction of light with materials greatly help in the understanding of what could be the medieval design of the symbolic of light. Such a gold-guiding of the natural light to reduce the cast shadows behind the statues could be useful to enhance the electric lighting of today for our monuments at dark hours.

7. Acknowledgements

The authors want to thank Gregory Lepère from the MENSI-Trimble company for his great availability during the TerraNumerica project and for giving them the useful 3D data on *Notre-Dame de Paris*.

References

[Abe63] ABELES F.: Methods for determining optical parameters of thin films. In *Progress in optics* (1963), Wolf E., (Ed.), North Holland Publishing Co., Amsterdam, pp. 249–288. 6

[Bri80] BRILL T. B.: *Light. Its Interaction with Art and Antiquities*. Plenum-Press, New-York and London, 1980. 4

[Cal96] CALLET P.: Pertinent data for modelling pigmented materials in realistic rendering. *COMPUTER GRAPHICS* forum *15*, 2 (1996), 119–127. 4

[Cal05] CALLET P.: Measured, calculated and computed color - physics and computer graphics -. In *Actes du colloque IRIS 2005* (2005), Hardy J., (Ed.), ENSAIA. 4

[CCCA73] COHEN R., CODY G., COUTTS M., ABELES B.: Optical properties of granular silver and gold films. *Physical Review B8*, 3689 (1973). 6

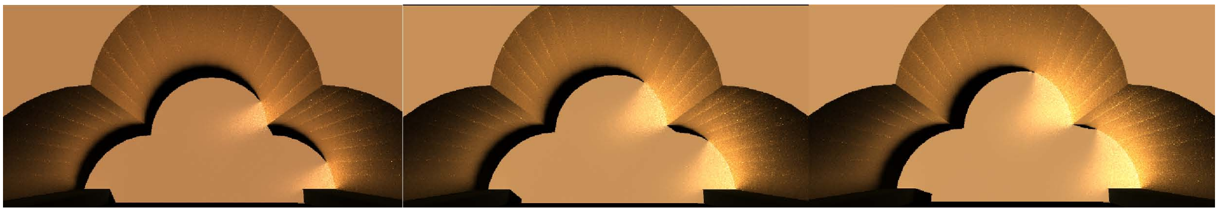

Figure 9: *Schematic evolution of the back ligthing of the heads kings due to caustics formation by reflection of the solar light at sunset. Light is guided by a grazing and progressive reflection on the gilded cylindrical shapes of the arcatures. From left to right the solar spectrum slightly changes according to the spectrophometric emission measurements made from 21:09 to 21:33 on 2009 july 15th.*

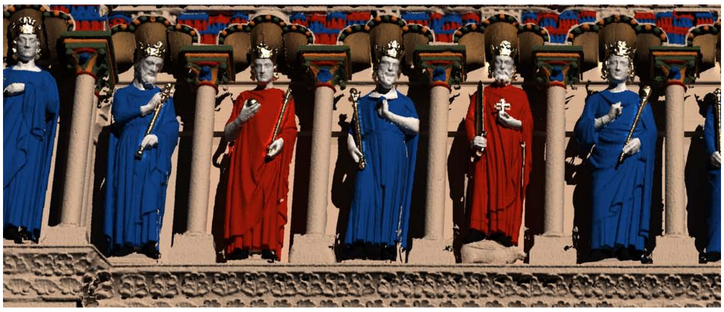

Figure 10: *Simulation obtained with Virtuelium using the photon-mapping algorithm, the optical model for gilts (see eq. 7) the polychromy indications available. Notice that the red and blue clothes are arbitrarily distributed here. The colours of any other ornaments are rendered accordingly to the historical indications [Fon00] given in Figure 5.*

[CS01] CALLET P., SÈVE R.: From mean diffuse external reflectance to color and visual appearance representation. In *AIC'01 Proceedings* (2001), SPIE. AIC 01 congress - Rochester, USA. 4

[DAGdC*07] DUMAZET S., ARIANE GENTY A. A. Z., DE CONTENCIN F.-X., TEXIER A., RUSCASSIER N., BONNET B., CALLET P.: Influence of the substrate colour on the visual appearance of gilded sculptures. In *Proceedings of the XXIth CIPA conference, Athens* (2007), Ogleby C., (Ed.), ISPRS. 4

[dSv55] DE SAINT-VICTOR H.: De tribus diebus. In *Patrologia Latina*. J.-P. Migne, Paris, 1844-1855. 3

[EB75] ERLANDE-BRANDENBURG A.: *Le Roi est mort, Etude sur les funérailles, les sépultures et les tombeaux des rois de France jusqu'à la fin du XIIIeme siècle, Paris*. Bibliothèque de la Société française d'archéologie,7, 1975. 3

[EB82] ERLANDE-BRANDENBURG A.: *Les sculptures de Notre-Dame de Paris au musée de Cluny*. Réunion des musés nationaux, 1982. 2, 4

[EB93] ERLANDE-BRANDENBURG A.: *Quand les cathédrales étaient peintes*. Collection Découvertes - Gallimard, 1993. 3

[Fon00] FONQUERNIE B.: Traces de la polychromie sur les portails et la galerie des rois de Notre-Dame de Paris. In *Actes du colloque : La couleur et la pierre. Polychromie des portails gothiques* (Paris, 2000), et Delphine Steyaert D. V., (Ed.), A. et J. Picard, pp. 119–128. 3, 4, 5, 8

[Gar04] GARNETT J. M.: Colours in metal glasses and in metallic films. *Philos. Trans. R. Soc. Lond. 203*, 385 (1904). 6

[GET] *Painting Materials*. 4

[Hal89] HALL R.: *Illumination and Color in computer Generated Imagery*. Springer-Verlag, New York, 1989. 7

[HM92] HAASE C. S., MEYER G. W.: Modeling pigmented materials for realistic image synthesis. In *ACM Transactions on Graphics* (1992), pp. 305–335. 4

[Jen97] JENSEN H.: Rendering Caustics on Non-Lambertian Surfaces. *Computer Graphics Forum 16*, 1 (1997), 57–64. 7

[Jen99] JENSEN H.: A Practical Guide to Global Illumination Using Photon Maps. *ACMSIGGRAPH 99* (1999), 1–72. 7

[NCNS03] NEUMANN L., CASTRO F., NEUMANN A., SBERT M.: Color appearance in multispectral radiosity. In *Proceedings on the 2nd Hungarian Computergraphics and Geometry Conference* (2003), L. Szirmay-Kalos G. R., (Ed.), pp. 183–194. 7

[Pas90] PASTOUREAU M.: La couleur et l'historien. In *Colloque International du CNRS. Pigments et colorants de l'Antiquité et du Moyen Age* (Paris, 1990), CNRS, (Ed.), Editions du CNRS, pp. 21–40. 3

[VIO56] *Dictionnaire raisonné de l'architecture française du XIe au XVIe siècle*. Bance, 1856. 3