

PCR: A Geometric Cocktail for Triangulating Point Clouds Beautifully Without Angle Bounds

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Abstract

Reconstructing a triangulated surface from a point cloud through a mesh growing algorithm is a difficult problem, in largely because they use bounds for the admissible dihedral angle to decide on the next triangle to be attached to the mesh front. This paper proposes a solution to this problem by combining three geometric properties: proximity, co-planarity, and regularity; hence, the PCR cocktail. The PCR cocktail-based algorithm works well even for point clouds with non-uniform point density, holes, high curvature regions, creases, apices, and noise.

CCS Concepts

•Computing methodologies → Mesh models; Point-based models;

1. Introduction

The surface reconstruction from a point cloud is an important research topic in computer graphics, geometric modeling, virtual reality, computer animation, medicine, and reverse engineering, just to mention a few. In the literature, we find several methods to surface reconstruction, which can be grouped in three main areas: simplicial surfaces, parametric surfaces, and implicit surfaces [GVJ*09]. The simplicial methods may also be subdivided in two groups: the ones that are based on the Voronoi/Delaunay techniques [DG03] and the mesh growing methods [XLC14].

The surface mesh reconstruction of a point cloud is a difficult task because of the possibly varying (or non-uniform) point density of the input point cloud, and because the shape of the original object may present holes, sharp features (e.g., creases and apices) and other high-curvature features, as well as noise. In this paper, our focus is on a surface mesh reconstruction method that follows the mesh growing strategy. Recall that mesh growing methods create a triangulated surface from an initial triangle [LHW09] [ASG11] [WTS12] [WZZW13] [XLC14], so that every single new triangle is attached to a mesh front's triangle in conformity with one or more geometric criteria. Our mesh growing algorithm, called PCR Cocktail, is based on three geometric properties, such as proximity (P), coplanarity (C), and regularity (R); hence, the PCR Cocktail algorithm here proposed.

Unlike other mesh growing algorithms, the PCR Cocktail algorithm does not impose bounds to the dihedral angle between the adjacent triangles nor angular restrictions on the internal angles of each mesh triangle. Nevertheless, PCR Cocktail produces meshes

whose triangles are tendentiously regular, i.e., the triangles tend to be equilateral. Therefore, there is no need for a regularization step after the mesh construction. Furthermore, it is not sensitive to point density of the input point cloud. Also, its triangulation flows from low curvature regions to high curvature regions, i.e., it gives priority to coplanar or low curvature regions in the triangulation of a given point cloud. This sort of curvature-based mesh segmentation allows us to control the correctness of the final mesh, even in presence of noise in the mesh.

2. PCR Cocktail Algorithm: Overview

Essentially, our algorithm consists of the following two main steps:

- *k-nearest neighbors*. Compute the k nearest neighbors of each point.
- *Mesh growing*. Build a triangulation from cloud points and their neighbors in an incremental manner.

Our surface reconstruction algorithm is based on three fundamental geometric criteria: proximity, regularity, and coplanarity. The *proximity* is the geometric concept underlying the computation of the k -nearest neighbors of each point of the cloud. The value of k was determined empirically as 12, which seemingly is the most adequate for both dense and sparse (i.e., non-uniform) point clouds.

The *regularity* of the mesh has to do with the generation of approximately regular triangles, i.e. approximately equilateral triangles. We use a regularity function to determine how much regular each triangle is. Thus, the regularity criterion has to do with the

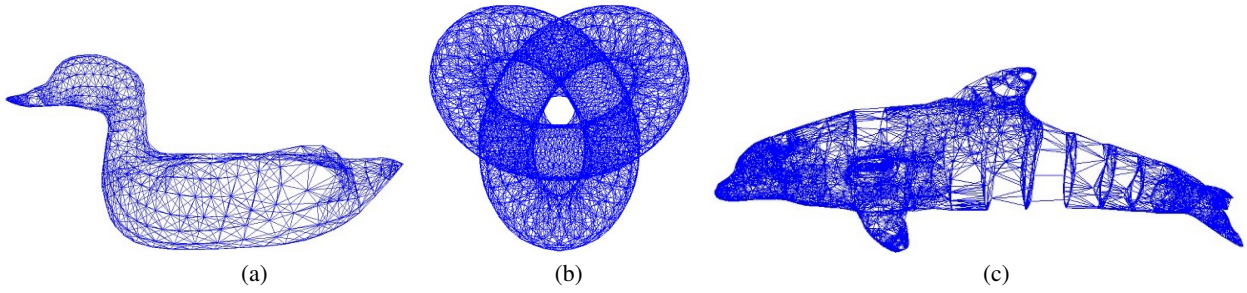


Figure 1: Proximity skeleton for three distinct point clouds using 12-nearest neighbors: (a) duck; (b) loop; (c) dolphin.

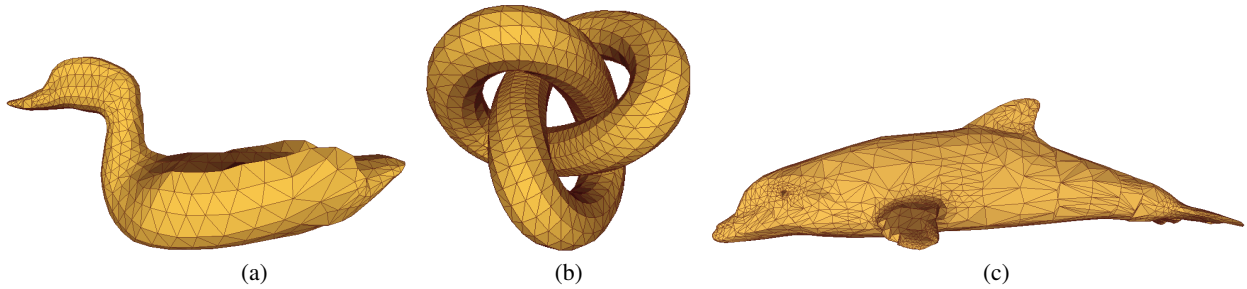


Figure 2: Reconstructed surface meshes using PCR Cocktail: (a) duck; (b) loop; (c) dolphin.

triangulation, and enters in action when we need to choose a triangle out of two putative triangles during a mesh growing step. More specifically, triangle T_1 is said to be more regular than triangle T_2 if the sum of the cosines of the three internal angles of T_1 is greater than the one of T_2 . This sum is here called *regularity function*, which is given by the following expression:

$$r_T(\alpha, \beta, \gamma) = \cos(\alpha) + \cos(\beta) + \cos(\gamma) \quad (1)$$

where α , β , and γ stand for the internal angles of a given triangle T . Note that this sum has a maximum of 1.5 (i.e., maximum regularity) when the three angles are identical and equal to 60° .

The *coplanarity* criterion is essentially used in the mesh growing step. The triangle to be added to the mesh front is the most coplanar with its nearest front triangle; hence, our argument of not using angular bounds. This means that triangles of sharp features are only triangulated after low curvature regions; the same applies to noisy regions. Another consequence of the priority given to low curvature regions in the mesh growing is to make away eventual mesh drifting effects (cf. Figure 1(b) and Figure 2(b)). Besides, by combining coplanarity and proximity, the PCR Cocktail algorithm is capable of successfully dealing with point clouds with variable point density (cf. Figure 1(c) and Figure 2(c)). Finally, our algorithm constructs a tendentiously regular triangle mesh, that is, triangles tend to be equilateral. This makes unnecessary any regularization step after the mesh construction.

3. Conclusions

We have introduced the PCR Cocktail algorithm, a surface mesh reconstruction algorithm that belongs to the family of mesh-growing algorithms. Our algorithm combines three geometric properties,

namely proximity, regularity and coplanarity, in order to decide which is the next triangle to be attached to the growing mesh front. Low curvature regions or even planar regions are triangulated before high curvature regions. To the best of our knowledge, no other mesh growing algorithm reunites these properties in an integrated manner. Even algorithms of other categories hardly enjoy these properties simultaneously.

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