A Sharpness Dependent Approach to 3D Polygon Mesh Hole Filling

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Abstract

A sharpness dependent hole filling approach is proposed in this paper. The proposed method can fill the hole of a mesh-based model and recover its sharp feature located at the hole area. Interpolation based on radial basis function is applied to create a smooth implicit surface and this surface can approximate the shape of the missed data. Then, a regularized marching tetrahedral algorithm is adopted to triangulate the above implicit surface and to produce a polygonal hole patch. A repaired mesh model is obtained by stitching the hole patch and the hole boundary of the original model. Finally, a feature enhancement process is applied if there exists a sharp feature on the hole boundary of the original model. The introduction of a sharpness dependent filter enhances the sharp feature of a hole-filled model. Experiment results show that our approach can produce excellent reparation results especially for recovering sharp feature.

1. Introduction

The reparation of incomplete polygonal mesh or the hole filling is a fundamental problem in computer graphics for reconstruction surfaces from the 3D scanned data of a given model, in which the scanned data cause some missing surfaces for the given model. The technique of hole filling in general is to keep the filled surface be continuously and smoothly fitted at the boundary of the hole in such a way that the original model shape be recovered. Recently, there are a few hole filling methods being proposed, in particularly the interpolation using volumetric technique [DMGL02, NT03] and Radial Basis Function for surface interpolation [CFB97, CBC*01] to reconstruction the missing surface are most often mentioned in literature. However, the technique of surface interpolation can render out only a smoothed model, which often with fine structure lost, because low order surface interpolation functions cannot produce sharp edge within the interpolation surface (Figure 1c).

By observing the shape structure of an incomplete data, the hole as shown in Figure 1a contains an edge for the missing surface. Therefore, when taking the polygonal mesh of the interpolated surface and of its nearby polygonal mesh of Figure 1c together into consideration, the polygon vertices of the interpolated surface need be readjusted to form an edge if the desired missing surface is to be recovered. This readjustment can be obtained from a filter design. For the polygon vertices of the interpolated surface can be regarded as vertices associated with sharp feature and the filter is to enhance the feature.

One way of recovering the sharp edge within a missing surface can be obtained by a specific filter design, called a sharpness dependent filter as proposed by the authors [CC05] for polygonal mesh. Suppose a given model is represented by a set of polygon faces including the hole is filled by a surface and the surface is meshed, then we consider a filter design for the set of local polygon faces which are within 2-ring distance of the hole. The filter design is to select the polygon face normal from the distribution of its neighboring face normals. Basically, the filter is formulated in such a way that if a polygon face has no sharp edge on its boundary, then the face normal tends toward the mean of its neighboring face normals; otherwise, it tends toward the closest normal of its neighboring faces. Hence, the sharp feature will extend from the hole boundary to the hole patch.

For an incomplete mesh model, it is required to detect the holes, fill the hole using ordinary surface interpolation method, mesh the surface that fills the hole and feed into the sharpness dependent filter described above to obtain a final

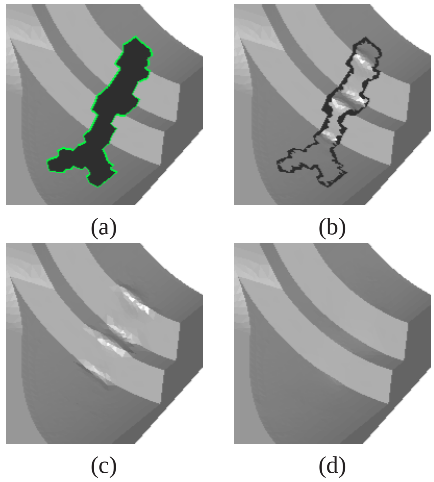


Figure 1: *Mesh model reparation. (a) the original mesh model with a hole; (b) the hole patch by using radial basis function interpolation; (c) the result of stitching the hole patch and the hole boundary; (d) the result obtained by applying the sharpness dependent filter hole filling approach.*

smoothed and recovered model. This paper is organized as follows: Section 2 gives detailed description of the proposed method for solving the problem of 3D polygon hole filling. Section 3 illustrates some examples to demonstrate the result of hole filling according to the proposed method. Section 4 draws the conclusion.

2. The Hole Filling Algorithm

The proposed hole filling algorithm is composed of the following steps: (1) fitting the hole by a smooth implicit surface according to the radial basis function interpolation method; (2) triangulating the implicit surface to obtain the polygonal meshes which fill the hole and stitching the polygonal meshes with original mesh model; and (3) applying the sharpness analysis to the polygon faces surrounding the hole and the meshes cover the hole for the recovery of the sharpness feature of the missing surface. In the following, we describe the proposed hole filling algorithm in detail.

2.1. Implicit surface fitting

The surface interpolation using radial basis function is popular in hole filling, because it needs not to arrange the data in order and needs only linear calculation. This approach is particularly suitable for the hole with irregular shape. By the same consideration, we adopt the radial basis function interpolation method to construct an implicit surface for filling the hole. The radial basis function surface is expressed as follows [YT02]:

$$f(x) = \sum_{i=1}^{n} d_i \psi(x - c_i) + P(x) \tag{1}$$

where $\psi(x)$ is a chosen radial basis function, c_i a control point, d_i a weight, and $P(x)$ the residual function of $f(x)$.

In order to solve the Equation (1), we specify the constrains of surface passing through the boundary vertices and their 2-ring neighbors as well as the surface normals on the 2-ring neighboring vertices [TO99]. Let $e_j = f(c_j)$ be a given constrain:

$$e_j = f(c_j) = \sum_{i=1}^{n} d_i \psi(c_j - c_i) + P(c_j) \tag{2}$$

then the constrain can be expressed in a matrix form as follows:

$$\begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1n} & 1 & c_1^x & c_1^y & c_1^z \\ \psi_{11} & \psi_{12} & \cdots & \psi_{1n} & 1 & c_2^x & c_2^y & c_2^z \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \psi_{n1} & \psi_{n2} & \cdots & \psi_{nn} & 1 & c_n^x & c_n^y & c_n^z \\ 1 & 1 & \cdots & 1 & 0 & 0 & 0 & 0 \\ c_1^x & c_2^x & \cdots & c_n^x & 0 & 0 & 0 & 0 \\ c_1^y & c_2^y & \cdots & c_n^y & 0 & 0 & 0 & 0 \\ c_1^z & c_2^z & \cdots & c_n^z & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \\ P_1 \\ P_2 \\ P_3 \\ p_4 \end{bmatrix}$$

$$= \begin{bmatrix} e_1 & e_2 & \cdots & e_n & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{3}$$

where $\psi_{ij} = \psi(c_i - c_j)$, e_i is chosen equal to 0 if the control point c_i is on the surface else $e_i \neq 0$, d_i and P_k are unknown coefficients to be determined.

In the proposed algorithm, the radial basis function $\psi(x) = \|x\|^3$ is chosen, because as mentioned in [Duc76], it can yield a smooth implicit surface that minimizes the curvature energy. The curvature energy function is defined as follows [TWBO03]:

$$E = \int\int (\kappa_{max}^2 + \kappa_{min}^2) dA \tag{4}$$

where κ_{max} and κ_{min} are the two principal curvatures of the surface and A is the area of the surface.

2.2. Surface triangulation

When an implicit surface is constructed, we need to mesh the surface as a collection of planar triangles in such a way that they can integrate with the original mesh model. We use regularized marching tetrahedral (RMT) method [CP98, TPG99] to perform the task of triangulating surface meshes. We use the shortest edge length of the hole as a unit size of the sampling cube surrounding the hole boundary. The surrounding cubes grow towards the inner part of the interpolated surface until the hole is completely filled with the sampled cubes. Then the RMT algorithm is applied to find all of the intersection points of the interpolated surface and the sampled cubes so that the surface patch of the hole is created.

Once the hole patch is created, we need to stitch the hole patch with the hole boundary of the original model to complete the polygon meshing for the missing surface. For each hole boundary vertex, the closest vertex on the hole patch boundary is considered as its correspondence vertex. The stitching is to merge the boundary vertices and their correspondence vertices. Assume b_i and b_{i+1} are two neighboring boundary vertices and p_k denote the vertex on the boundary of the hole patch, we have:

1. if b_i and b_{i+1} correspond to the same vertex p_k, then merge these three vertices to become a vertex (Figure 2a);
2. if b_i and b_{i+1} correspond to p_k and p_{k+1} respectively, and p_k, p_{k+1} are neighboring vertices, then merge b_i with p_k and b_{i+1} with p_{k+1} (Figure 2b);
3. if b_i and b_{i+1} correspond to p_k and p_{k+n} respectively, and there is p_j between p_k and p_{k+n}, then merge p_j and p_k if the distance between p_j, p_k is less than the distance between p_j, p_{k+n}, otherwise, merge p_j, p_{k+n} first and then merge b_i with p_k and b_{i+1} with p_{k+n} (Figure 2c).

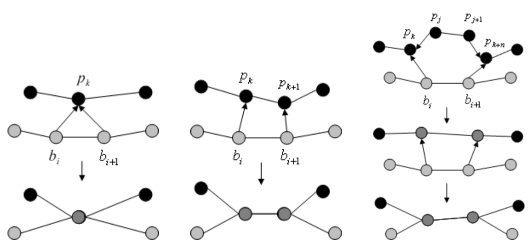


Figure 2: *The three vertex merging cases for stitch the hole patch and the hole boundary. The black points and the gray points denote the vertices on the boundary of hole patch and the vertices on the hole boundary. The deep gray points are the vertices after vertex merging.*

2.3. Feature enhancement

The final step of our hole filling algorithm is the process of feature enhancement. Before this process is applied, one has to determine whether a hole patch needs to apply this process. If the answer is negative, the hole filling algorithm terminates.

To determine whether a feature enhancement process is required is based on the distribution of the sharpness value of the triangle faces located in the 2-ring range of the hole boundary. The sharpness value of a triangle face is defined as the variance of the angle between its normal and the normals of its neighboring faces [CC05]. Using the Bayesian classification method, we are able to execute a two-class separation process [CCL04]. If the above algorithm can successfully separate the data into the sharp part and the non-sharp part, the feature enhancement process should be applied.

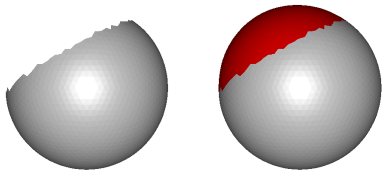
In this paper, we use sharpness dependent filter [CC05] to enhance the sharp feature of a hole patch. The concept of sharpness dependent filter is that if a face is located at the sharp feature area, then its face normal tends to the closest normal of its neighboring face. If a face is located at the smooth area, then its face normal tends to the mean of its neighboring face normals. Therefore, while there exists sharp feature surrounding a hole, the face normal of the hole patch should be readjusted so that it can be close to the face normal of a sharp region. In other words, the sharp features will extend from the hole boundary to the hole patch. Figure 1d shows the result of feature enhancement applying to the model in Figure 1c. The sharp edges of the model are recovered well.

3. Experiment results

In this section, we demonstrate some experiment results. First, we applied our algorithm to an incomplete sphere shown in Figure 3a. Using a curvature energy minimization radial basis function $\psi(x) = \|x\|^3$, we obtained an excellent hole filling result shown in Figure 3b. In Figure 4, we filled the hole of a bunny model. As can be seen in Figure 4, the hole patches and the original model were integrated smoothly. Figure 5 shows the result of filling the ear of a bunny model. In this case, we cut a small part of the bunny's ear and then repaired the hole via our algorithm. The result shows the proposed algorithm is indeed valid. Finally, we applied our algorithm to the hole with a corner. The reparation of a corner is a quite difficult problem for hole filling. Figure 6a shows our method could recover a convex corner. Figure 6b shows the result of applying our algorithm to a corner region which is the intersection of a convex edge and a concave edge.

4. Conclusions

Since the surface interpolation approach for 3D polygon hole filling can yield only a smoothly and continuously polygon fitting across the boundary of the missing surface, a sharpness dependent approach to 3D polygon hole filling is proposed to further guarantee the recovery of the original shape for a missing surface. The proposed 3D polygon hole filling method is based upon the classification of sharpness distributions of the polygon meshes, which surround the hole. If there exists sharp feature surrounding the hole, the polygon vertices are input to a sharpness dependent filter to diffuse the vertices until the sharp feature is completely recovered. Experiment result shows the sharpness dependent approach to 3D polygon hole filling can obtain excellent result for uniformly meshed missing surfaces.



(a) (b)

Figure 3: *The reparation result of an incomplete sphere model. (a) an incomplete sphere; (b) the reparation result, the hole patch is shown in highlight.*

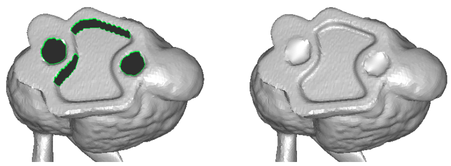
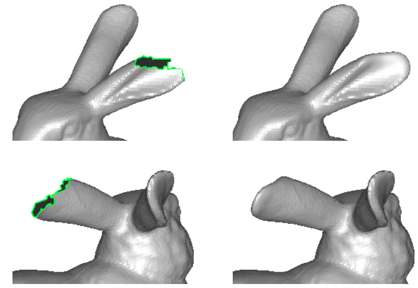
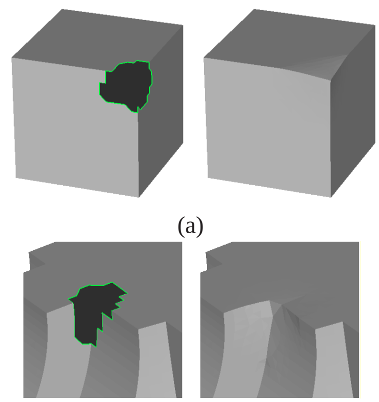


Figure 4: *The hole filling result of bunny model. The left one is the original model and the right one is the hole filling result.*



(a) (b)

Figure 5: *The hole filling result of bunny's ear. (a) the ear is cut and produce a hole; (b) the hole filling result obtained by applying the proposed hole filling method.*



(a)

(b)

Figure 6: *Applying the proposed hole filling algorithm to the hole with a corner. The left shows the original model and the right shows the result after hole filling. (a) the result of a convex corner; (b) the result of a corner which is the intersection of a convex edge and a concave edge.*

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