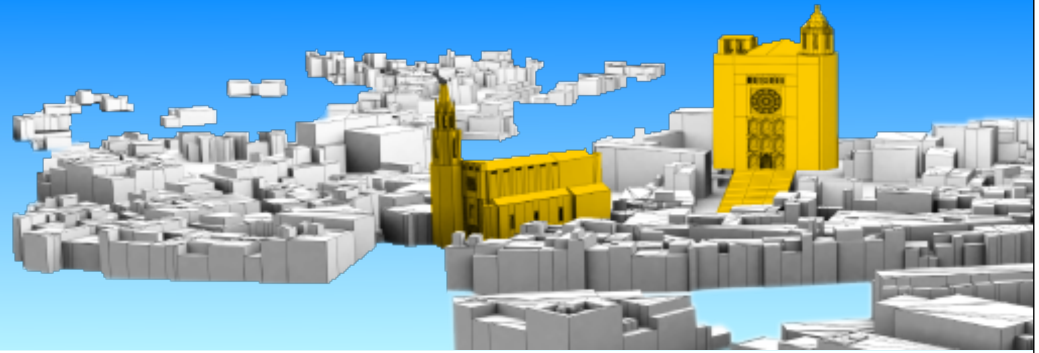
Eurographics 2013
May 6-10, Girona (Spain)



Tutorial

Symmetry in Shapes Theory and Practice

Niloy J. Mitra
University College London

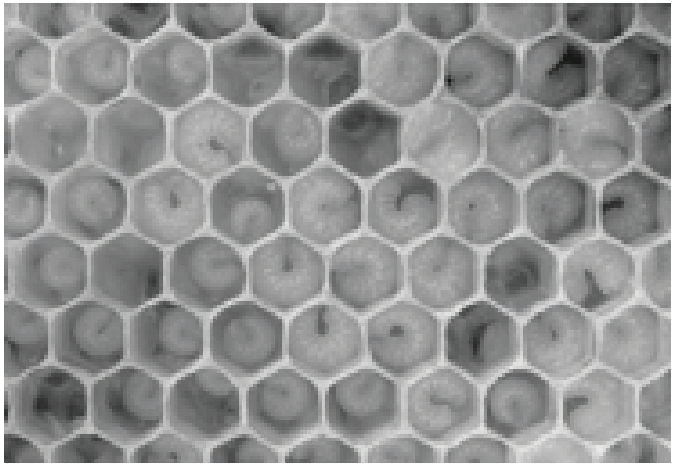
What we do *not* cover?

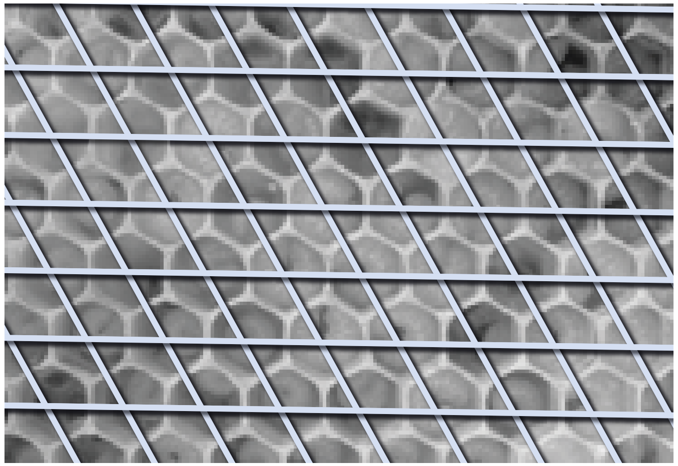
Symmetry detection on images

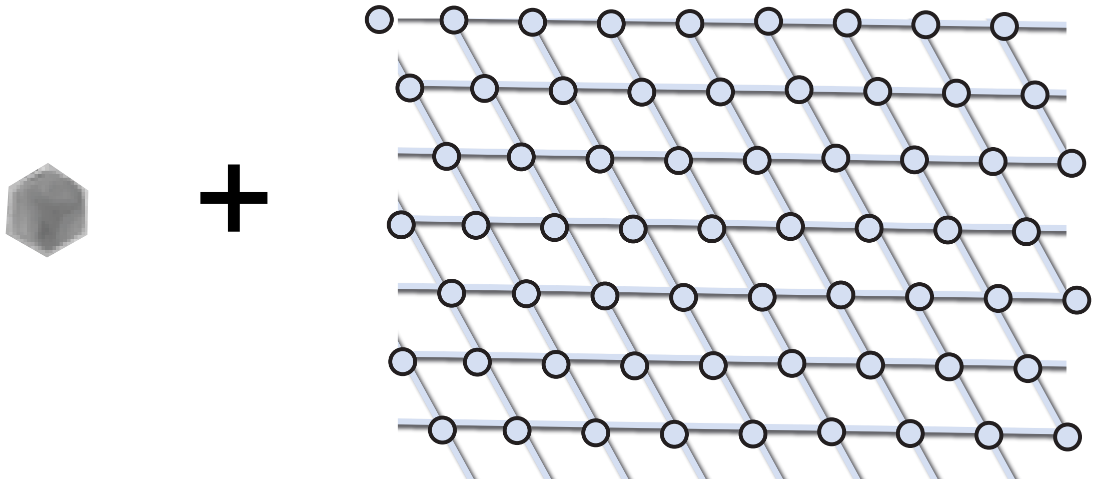
Global symmetry detection on 3D geometry

Intrinsic symmetry detection

Regular Structure







Problem Characteristics

Difficult

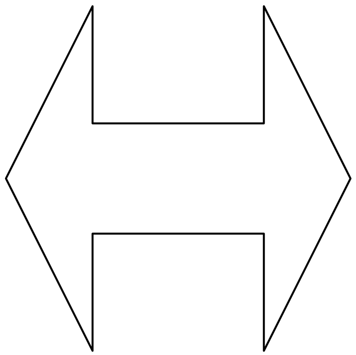
- Which parts are symmetric ⟶ objects are not pre-segmented
- Space of transforms: rotation, translation, scaling, etc.
- Brute force search is not feasible

Easy

- Proposed symmetries ⟶ easy to validate

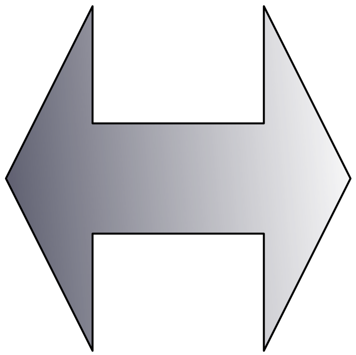
Symmetry Detection

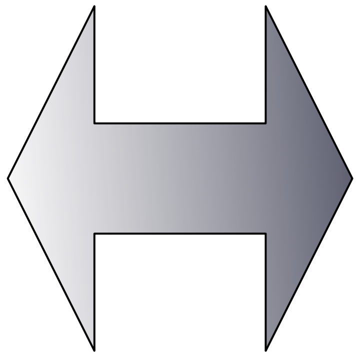
M



Geometric Matching

M_1 M_2

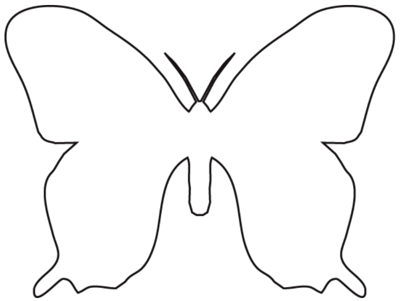


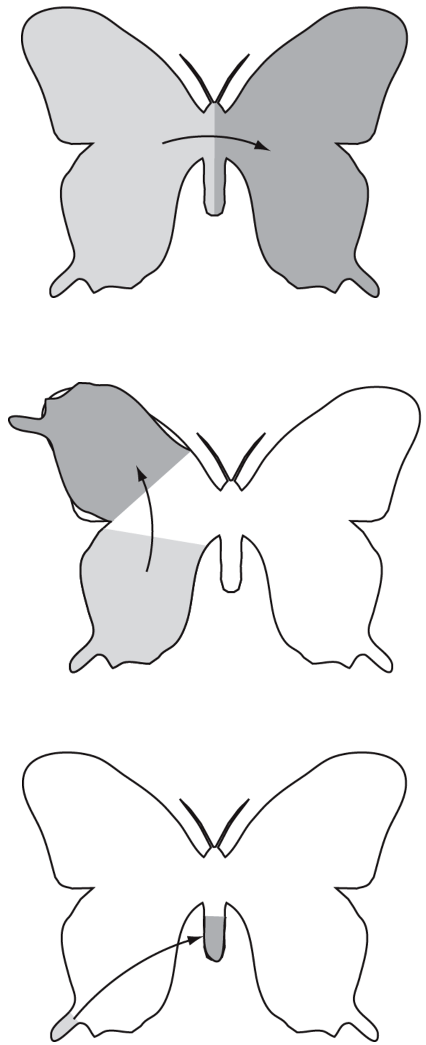


$$M_1 \approx T(M_2)$$

symmetry detection $M \approx T(M)$

Types of Symmetry





- Reflection
- Rotation + translation
- Uniform scaling

Typical Stages

- Feature selection

$$\mathcal{F}(M) = \mathcal{F}(T(M))$$

- Aggregation

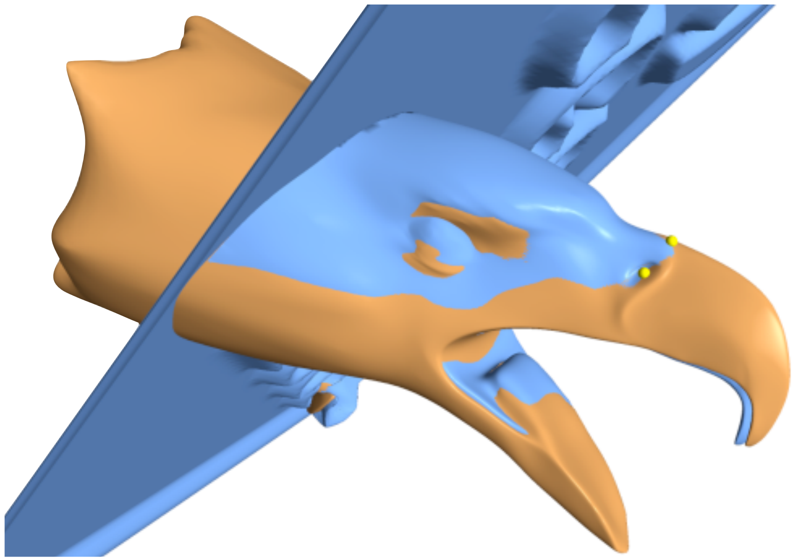
- Extraction

Geometric Hashing

Features: quadratic patch parameters

Aggregation: geometric hashing

Extraction: pre-segmentation



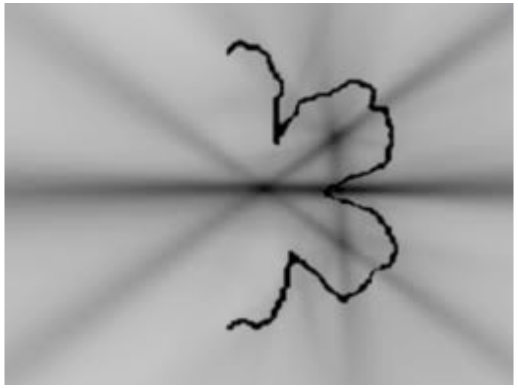
[Gal et al. 2006]

Symmetry Transform

Features:

Aggregation: FFT in transform domain

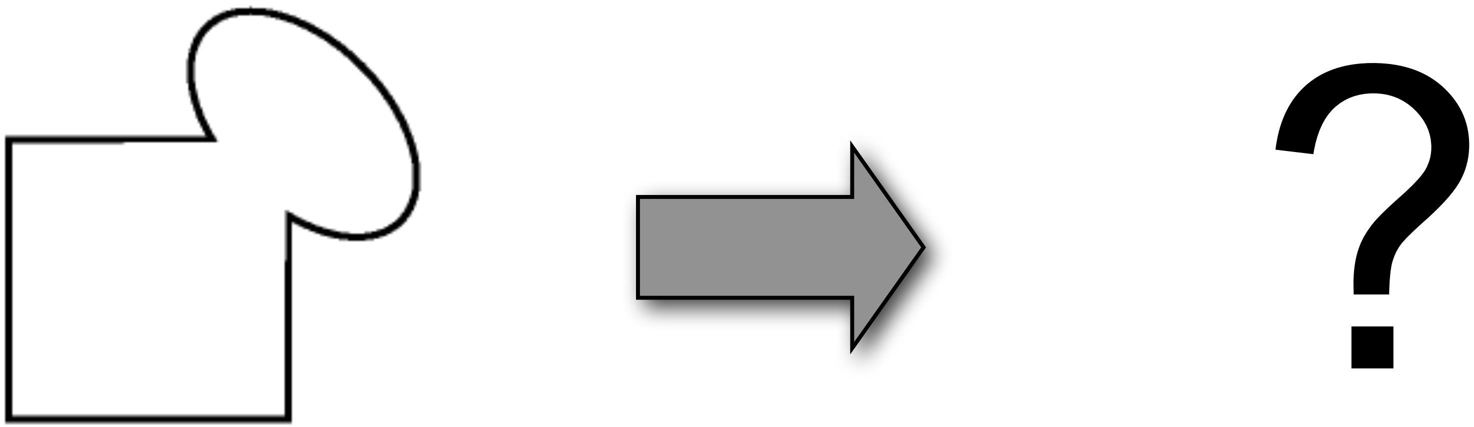
Extraction: clustering, region growing



[Podolak et al. 2006]

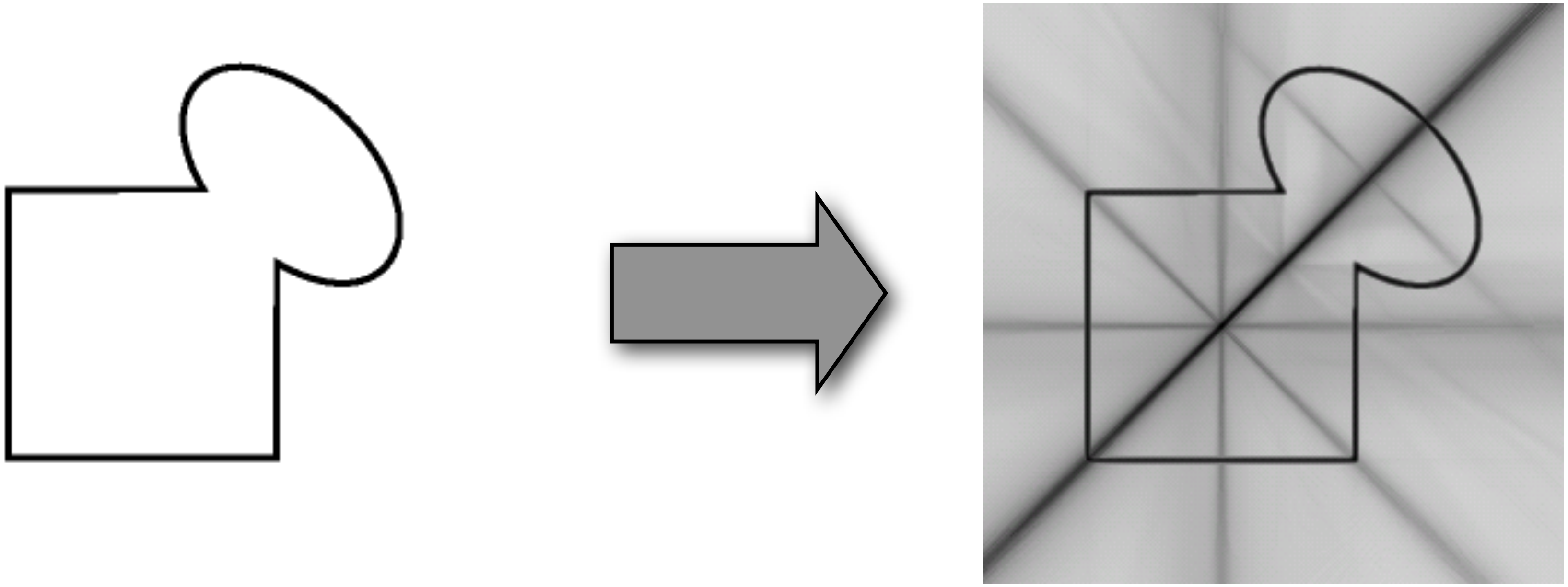
Goal

A computational representation that describes all planar symmetries of a shape



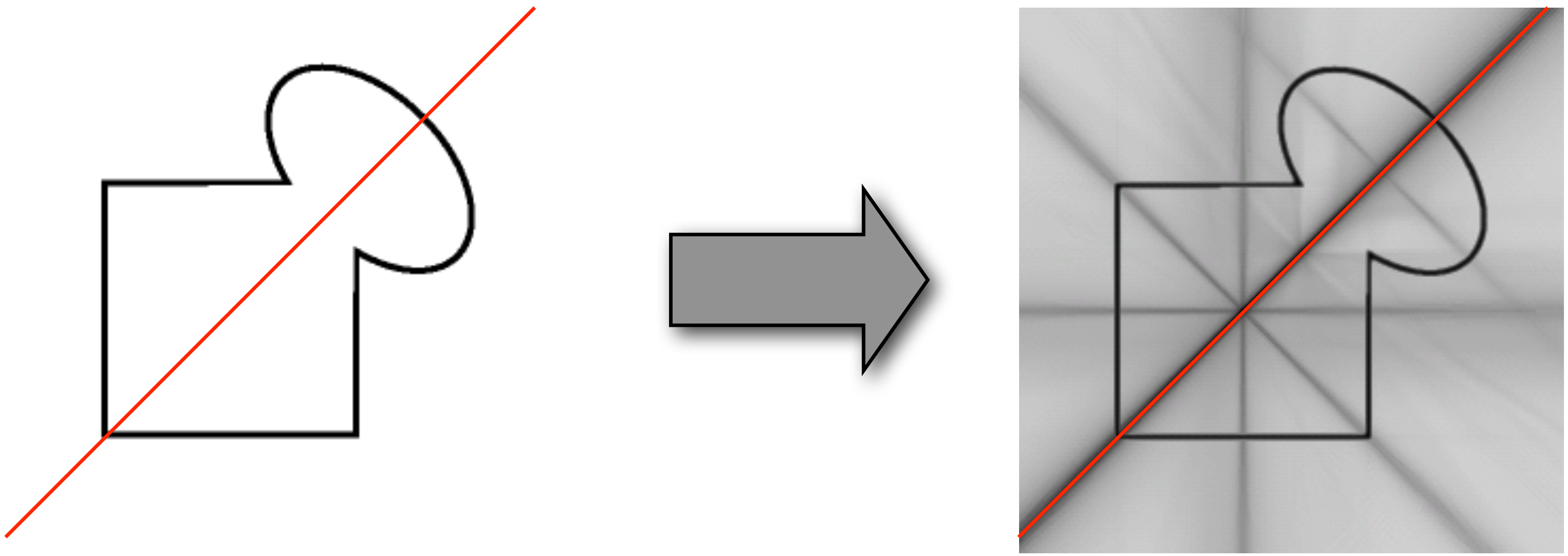
Symmetry Transform

A computational representation that describes all planar symmetries of a shape



Symmetry Transform

A computational representation that describes all planar symmetries of a shape

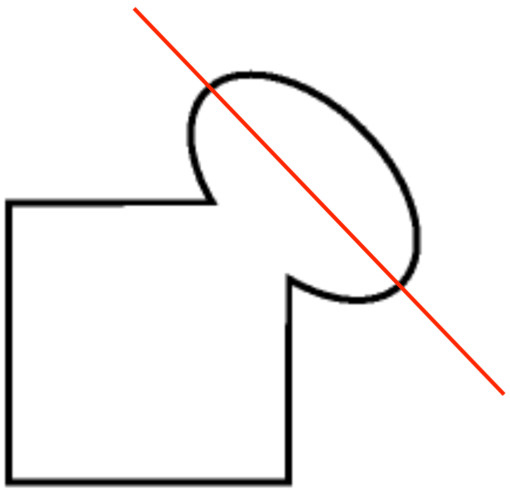


Perfect Symmetry

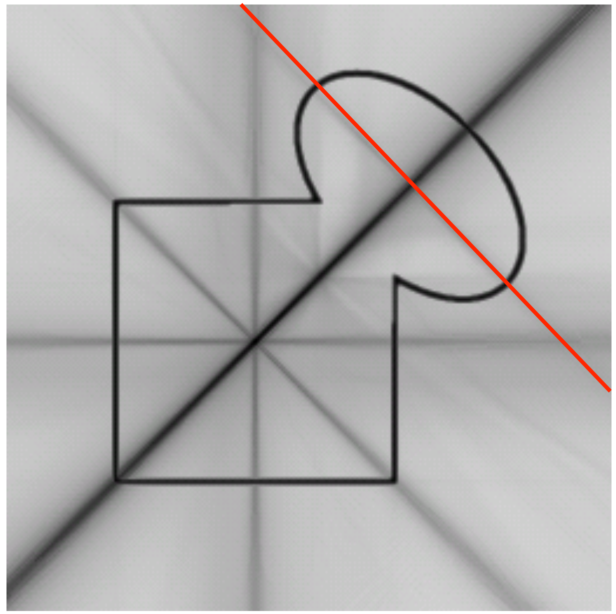
Symmetry = 1.0

Symmetry Transform

A computational representation that describes all planar symmetries of a shape



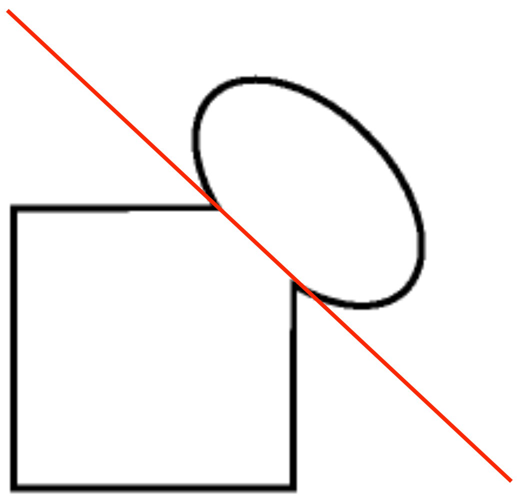
Local Symmetry



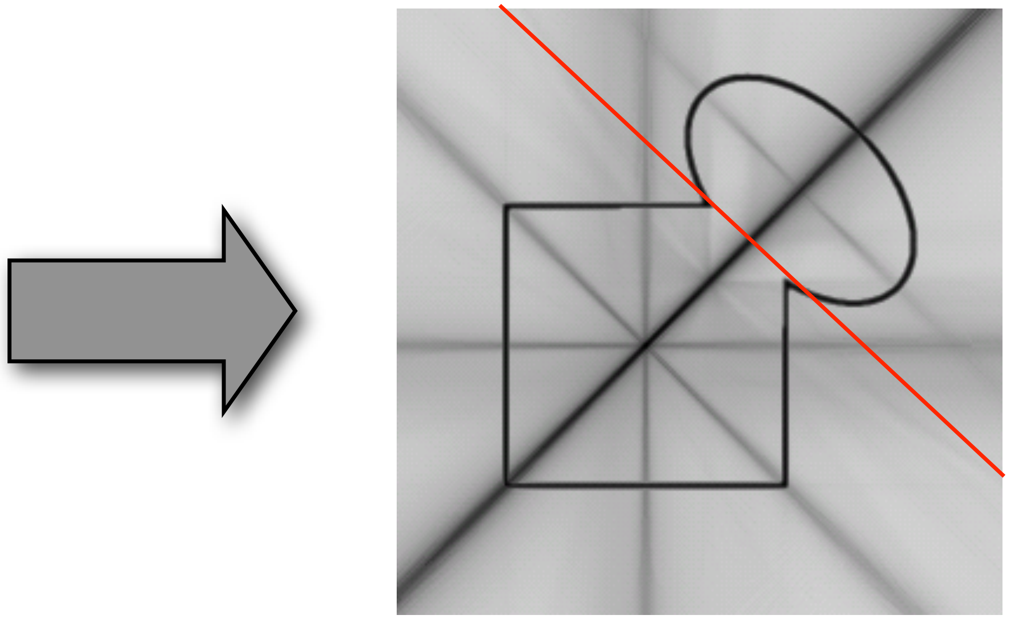
Symmetry = 0.3

Symmetry Transform

A computational representation that describes all planar symmetries of a shape



Partial Symmetry

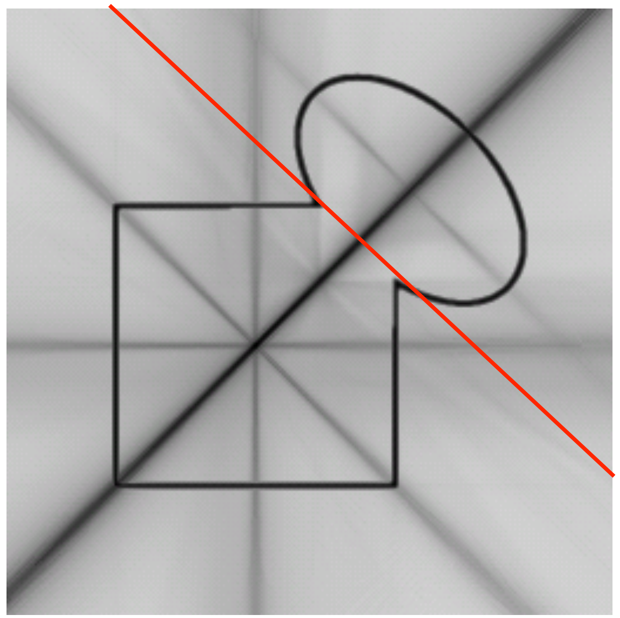


Symmetry = 0.2

Symmetry Transform

A computational representation that describes all planar symmetries of a shape

$$d(M,T) = \left\| \frac{M - T(M)}{2} \right\|$$



Symmetry = 0.2

Transform Domain Analysis

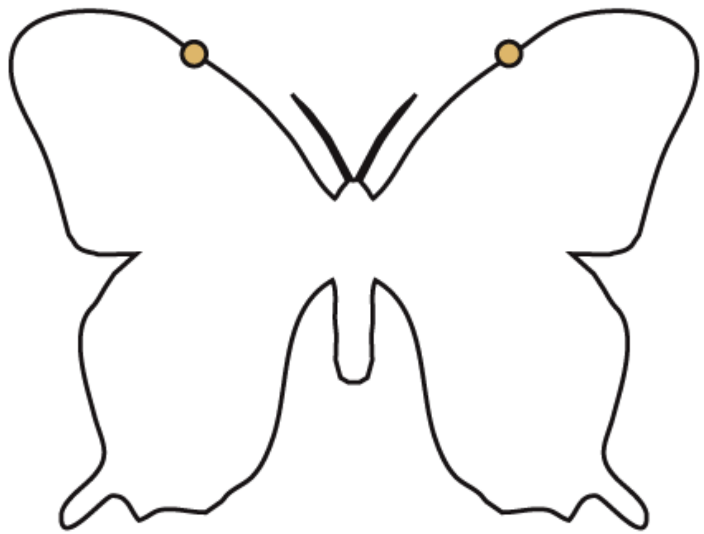
Features: curvatures

Aggregation: transform domain analysis

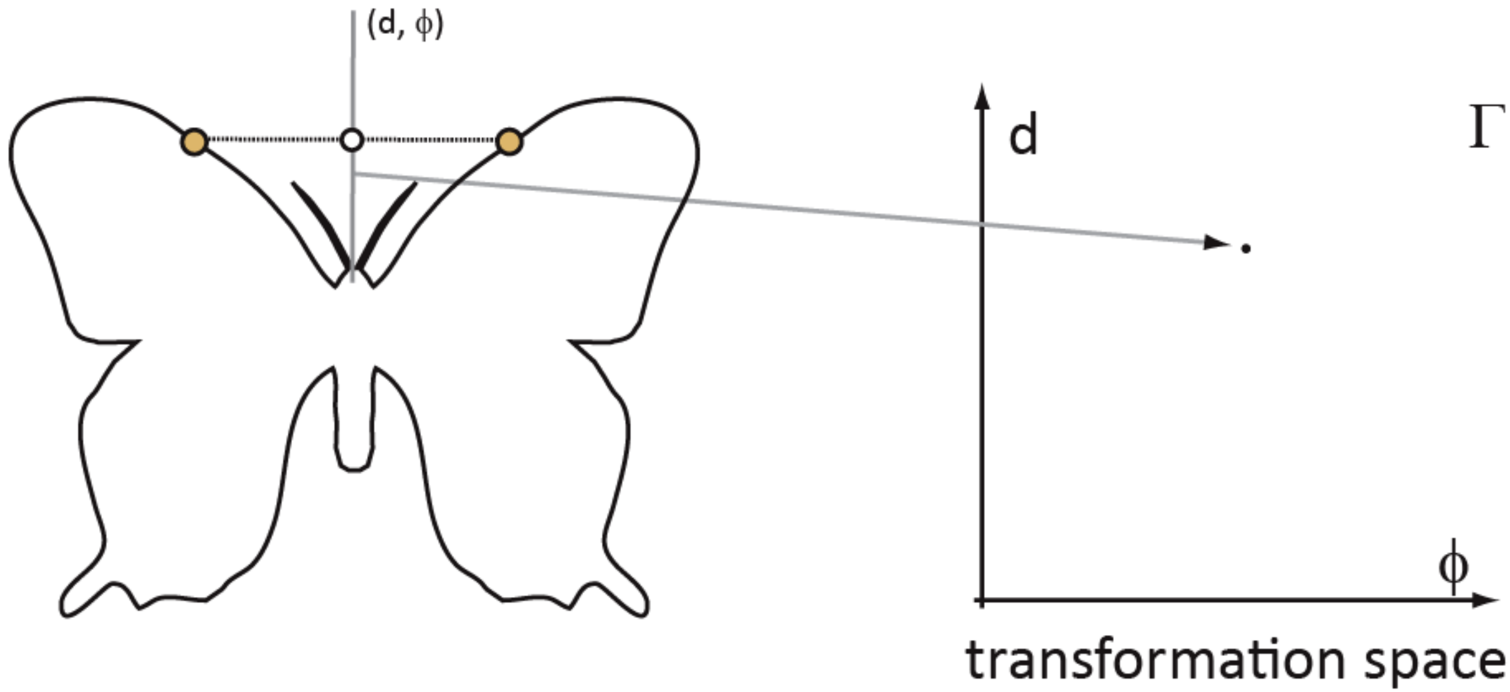
Extraction: region growing

[Mitra et al. 2006]

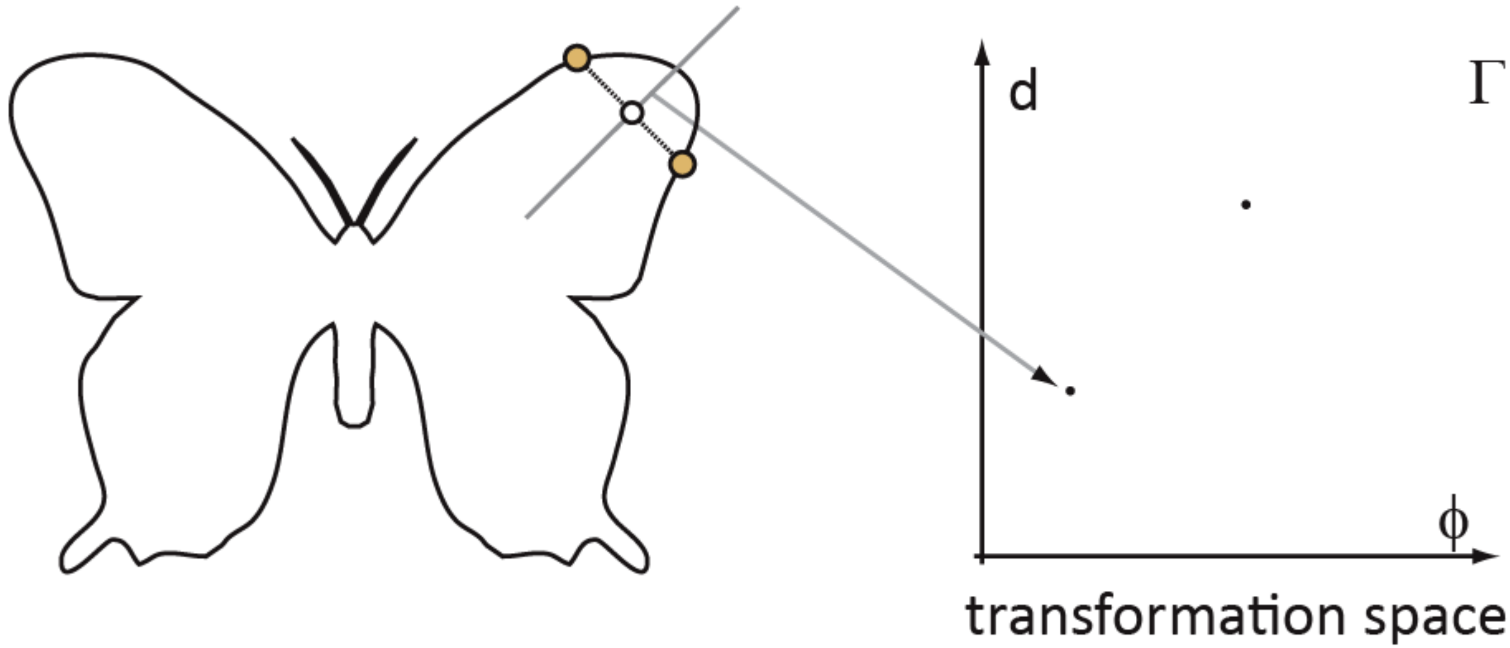
Reflective Symmetry



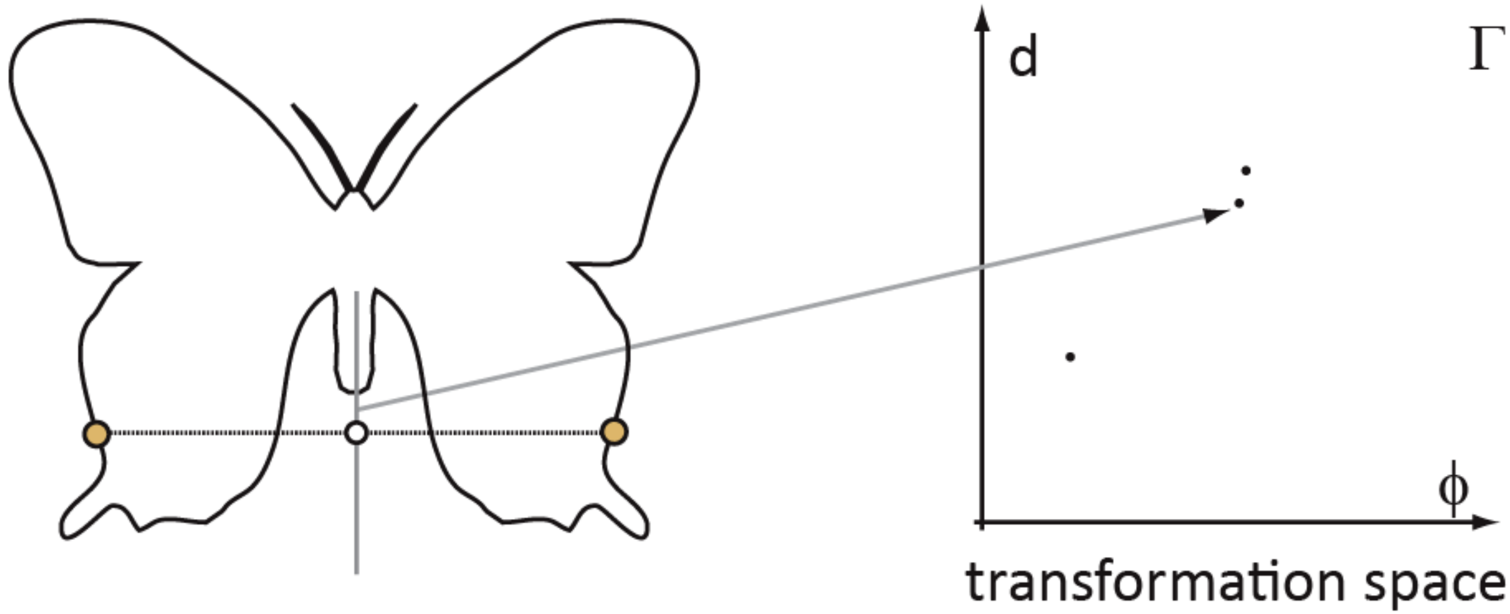
Reflective Symmetry: A Pair Votes



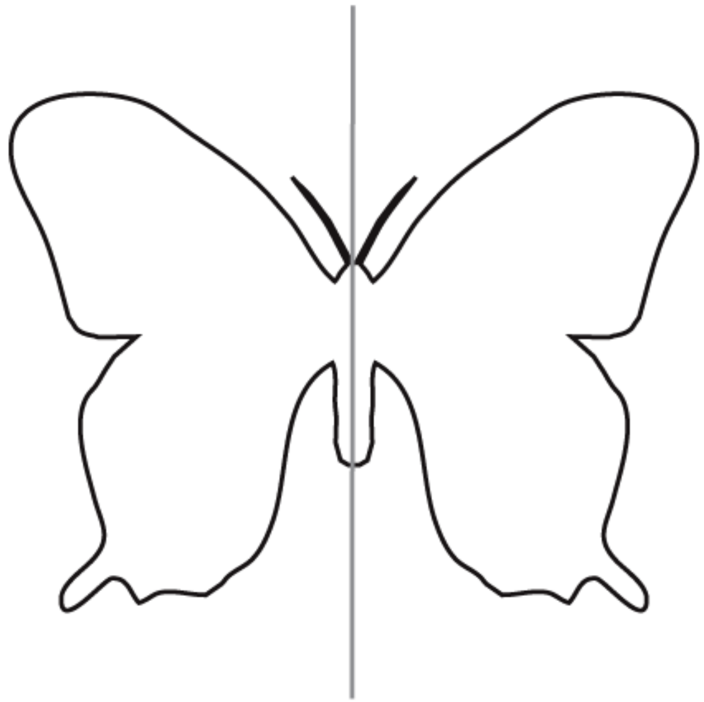
Reflective Symmetry: Voting Continues

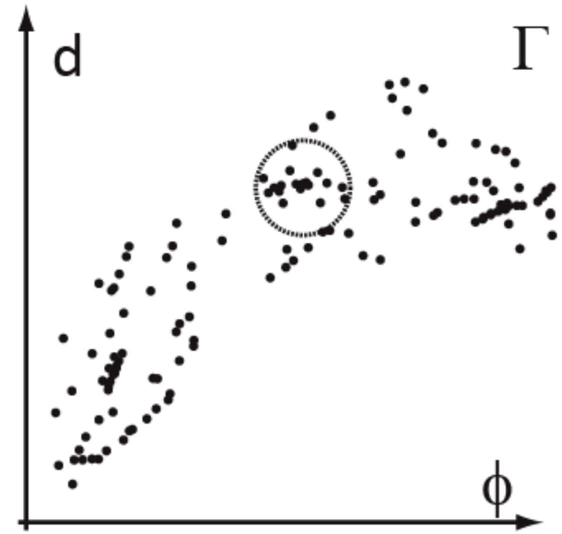


Reflective Symmetry: Voting Continues



Reflective Symmetry: Largest Cluster

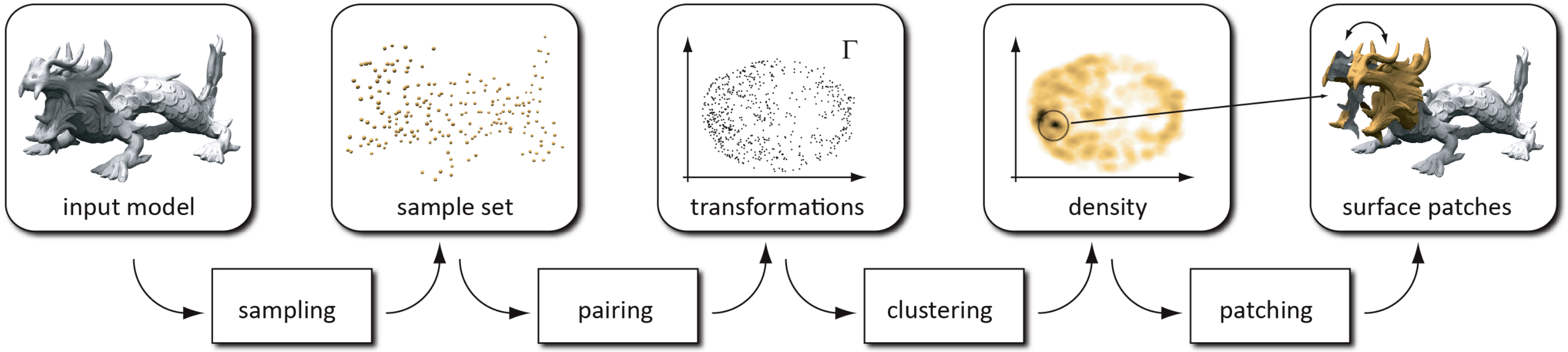




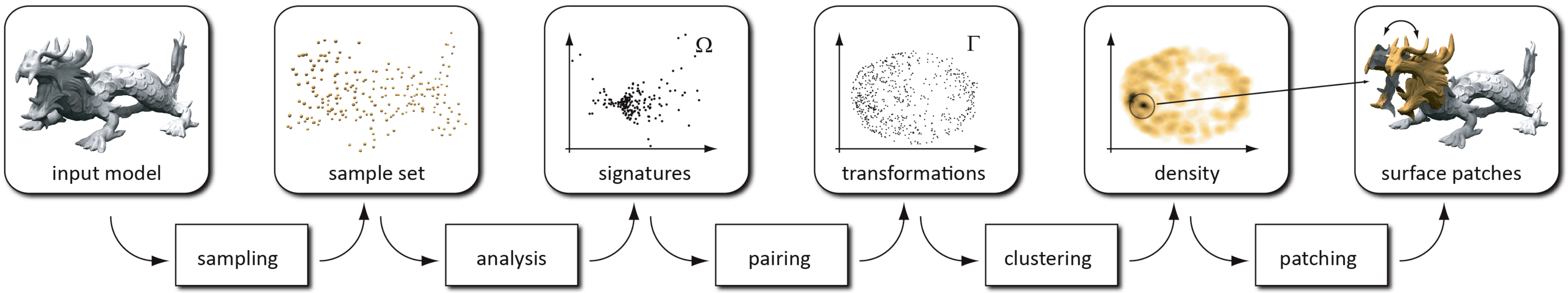
Height of cluster ⟶ size of patch

Spread of cluster ⟶ level of approximation

Pipeline

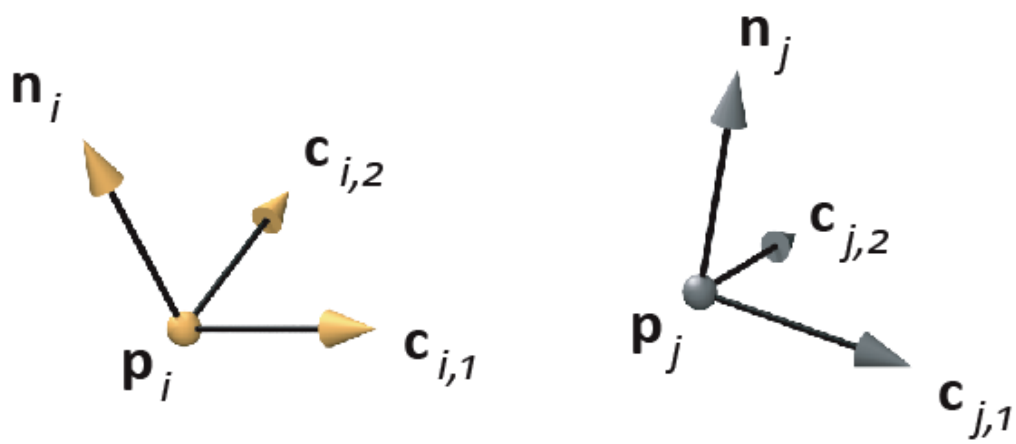

input model
sample set
transformations
Γ
density
surface patches
sampling
pairing
clustering
patching

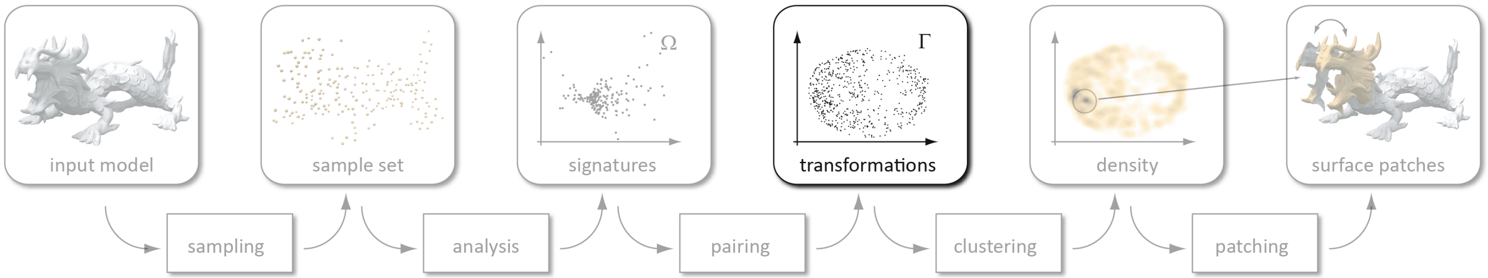
Pipeline



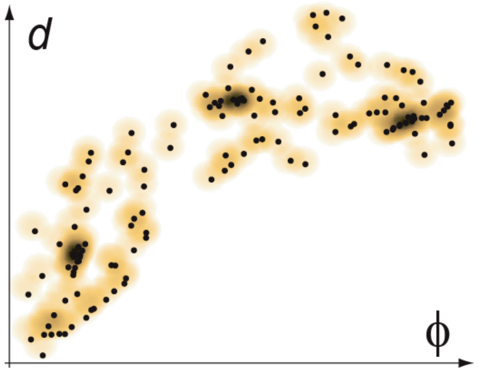
input model
sample set
signatures
Ω
transformations
Γ
density
surface patches
sampling
analysis
pairing
clustering
patching

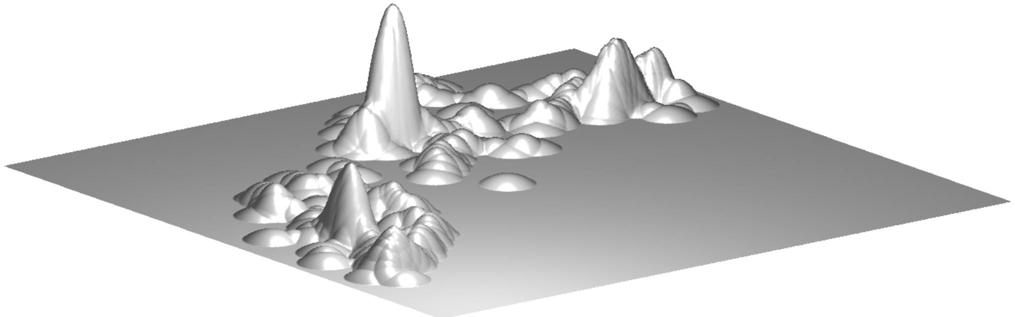
Rigid Transformations

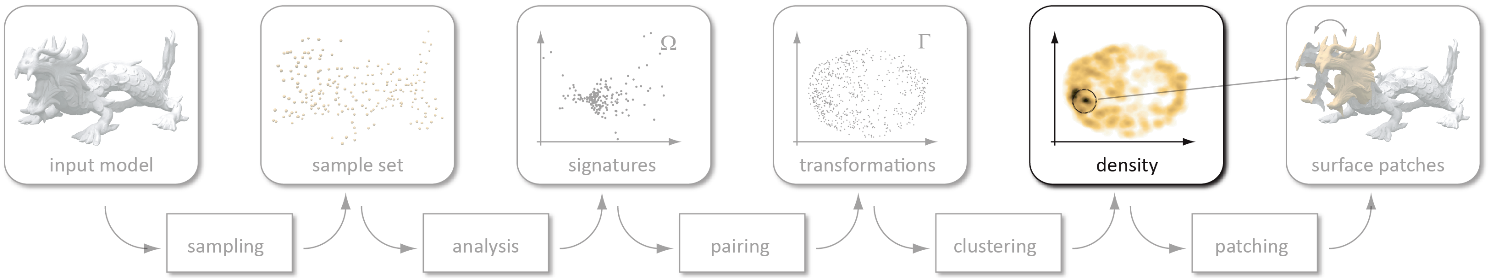




Mean-Shift Clustering

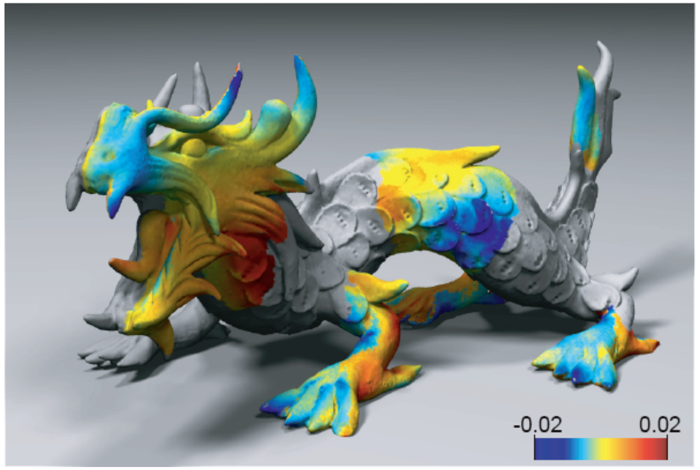






Detection Results: Dragon

detected symmetries



correction field

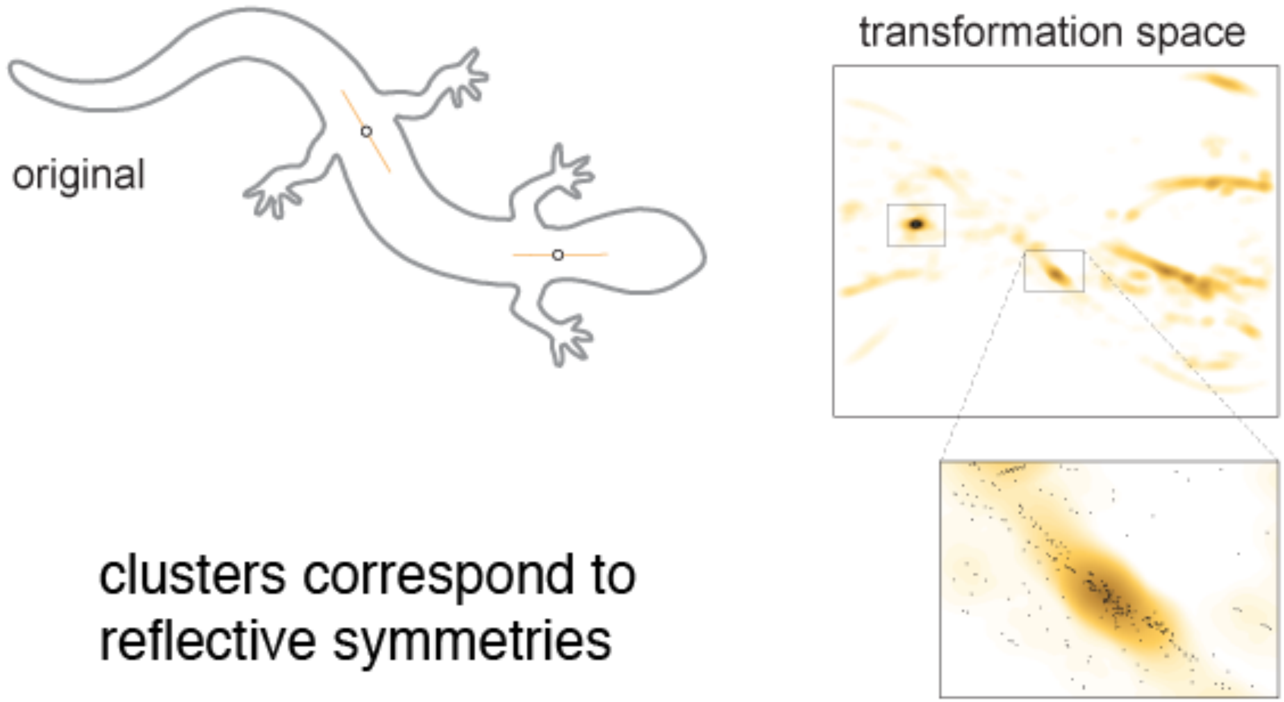
Insight: Global to Local Problem

(Euclidean) symmetry in spatial domain

↓

cluster(s) in transform domain

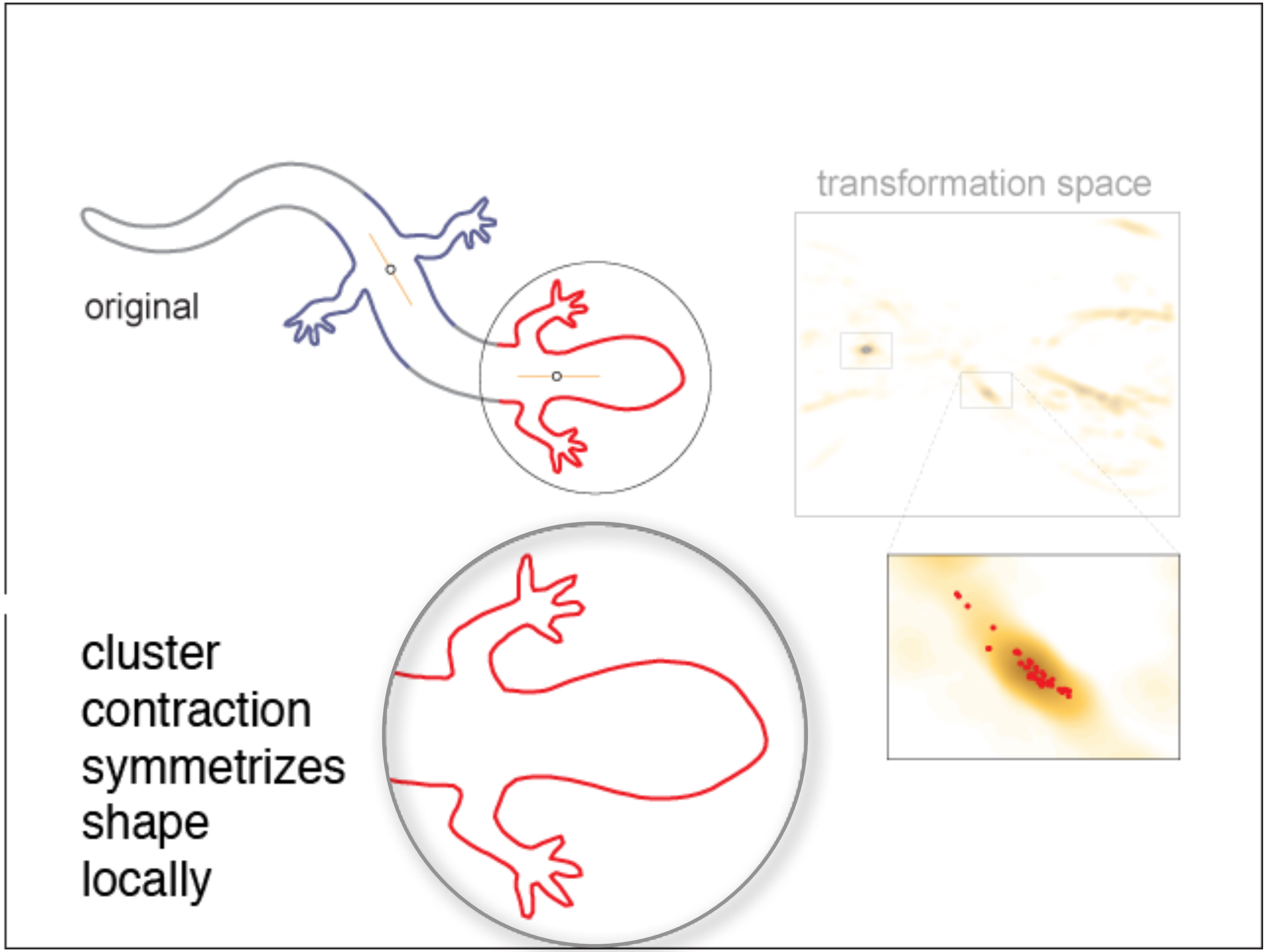
2D Example: Symmetrization



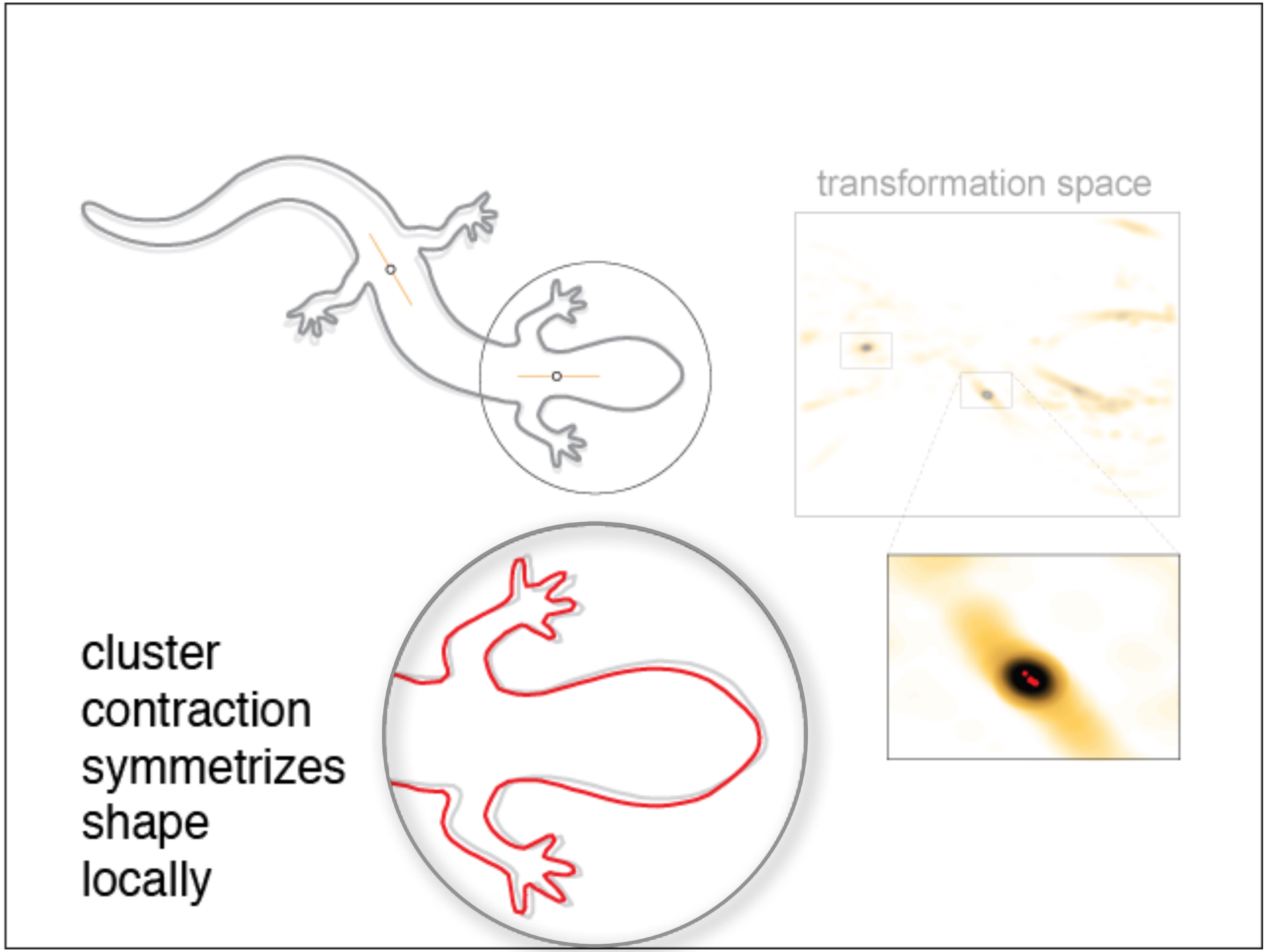
clusters correspond to
reflective symmetries

[Mitra et al. 2007]

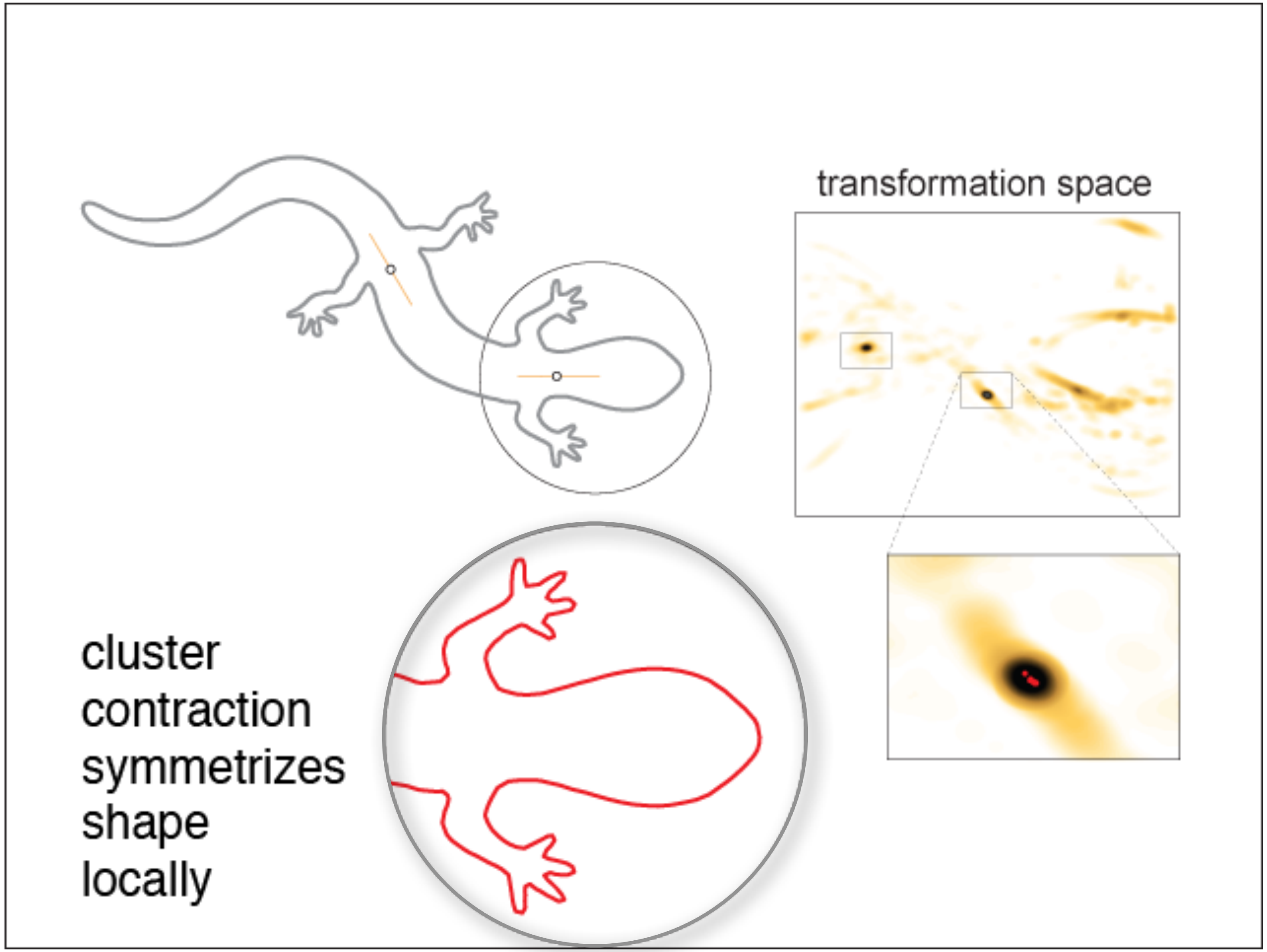
2D Example: Symmetrization



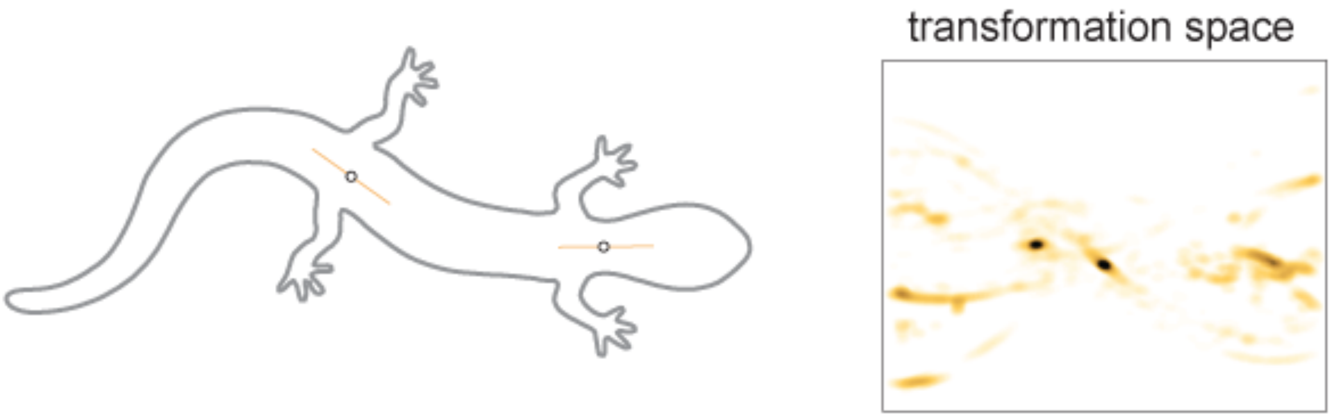
2D Example: Symmetrization



2D Example: Symmetrization

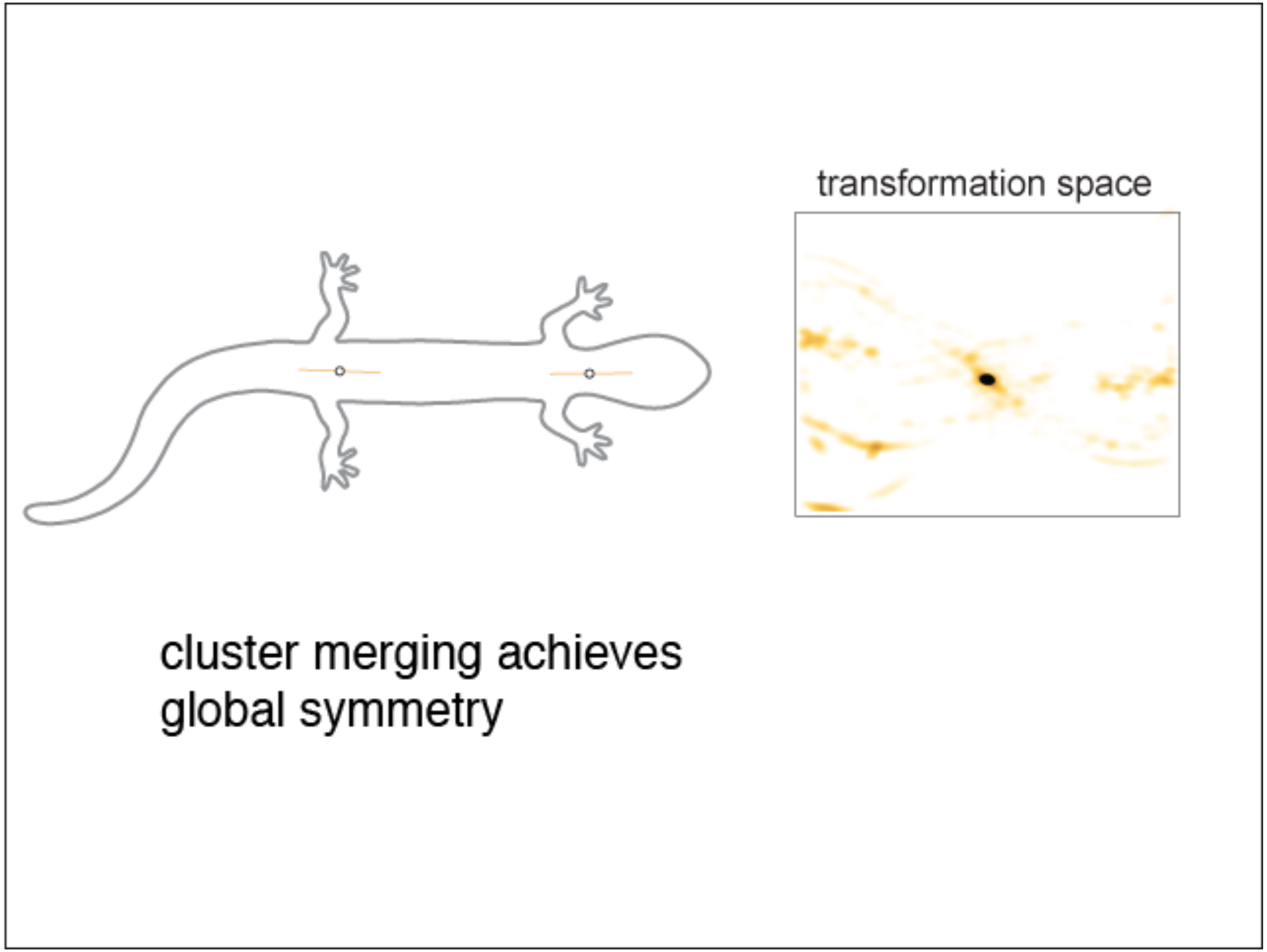


2D Example: Symmetrization

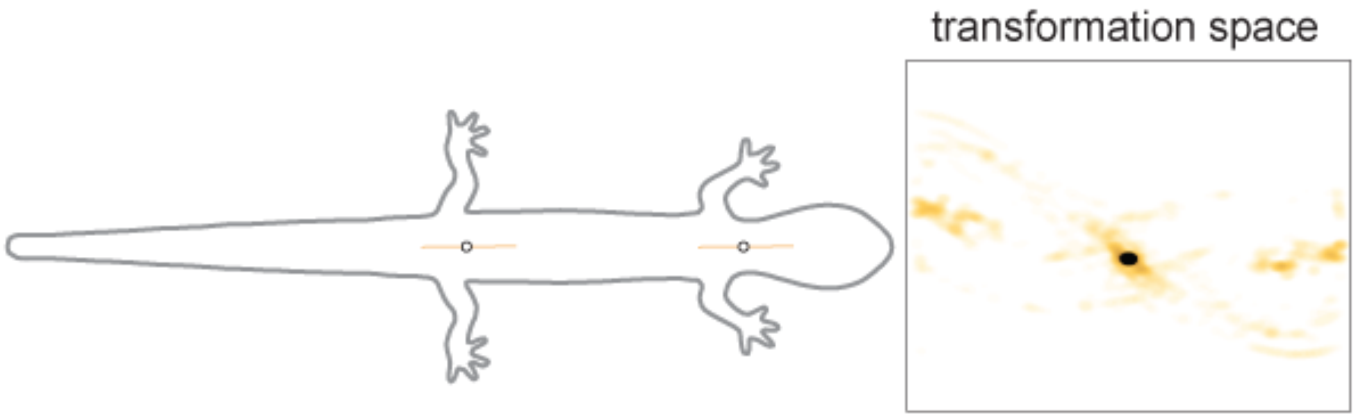


cluster merging achieves
global symmetry

2D Example: Symmetrization



2D Example: Symmetrization



cluster merging achieves
global symmetry

Symmetrization: Bunny

Cluster
Contraction

Transformation Space

Graph-based Symmetries

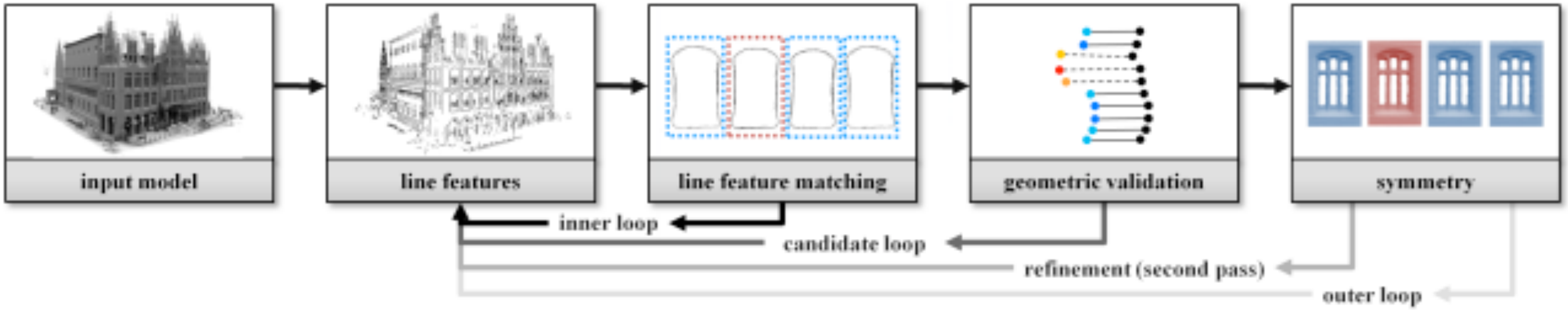
Features: slippage analysis

Aggregation: locally coherent line arrangements

Extraction: simultaneous refinement

[Bokeloh et al. 2009]

Algorithm Pipeline

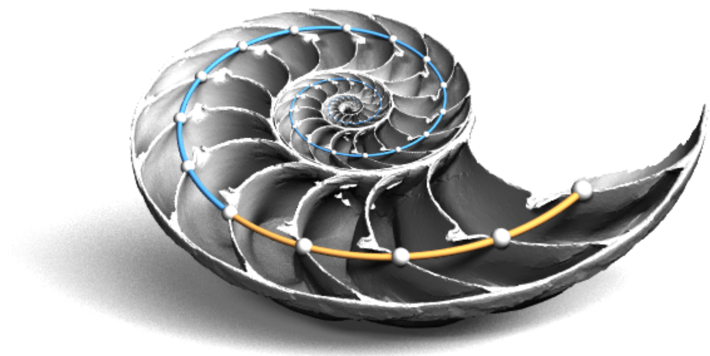


Symmetry of Symmetries

Features: curvatures

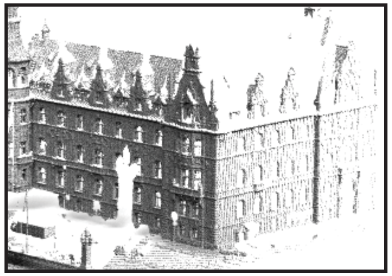
Aggregation: transform domain model extraction

Extraction: simultaneous refinement



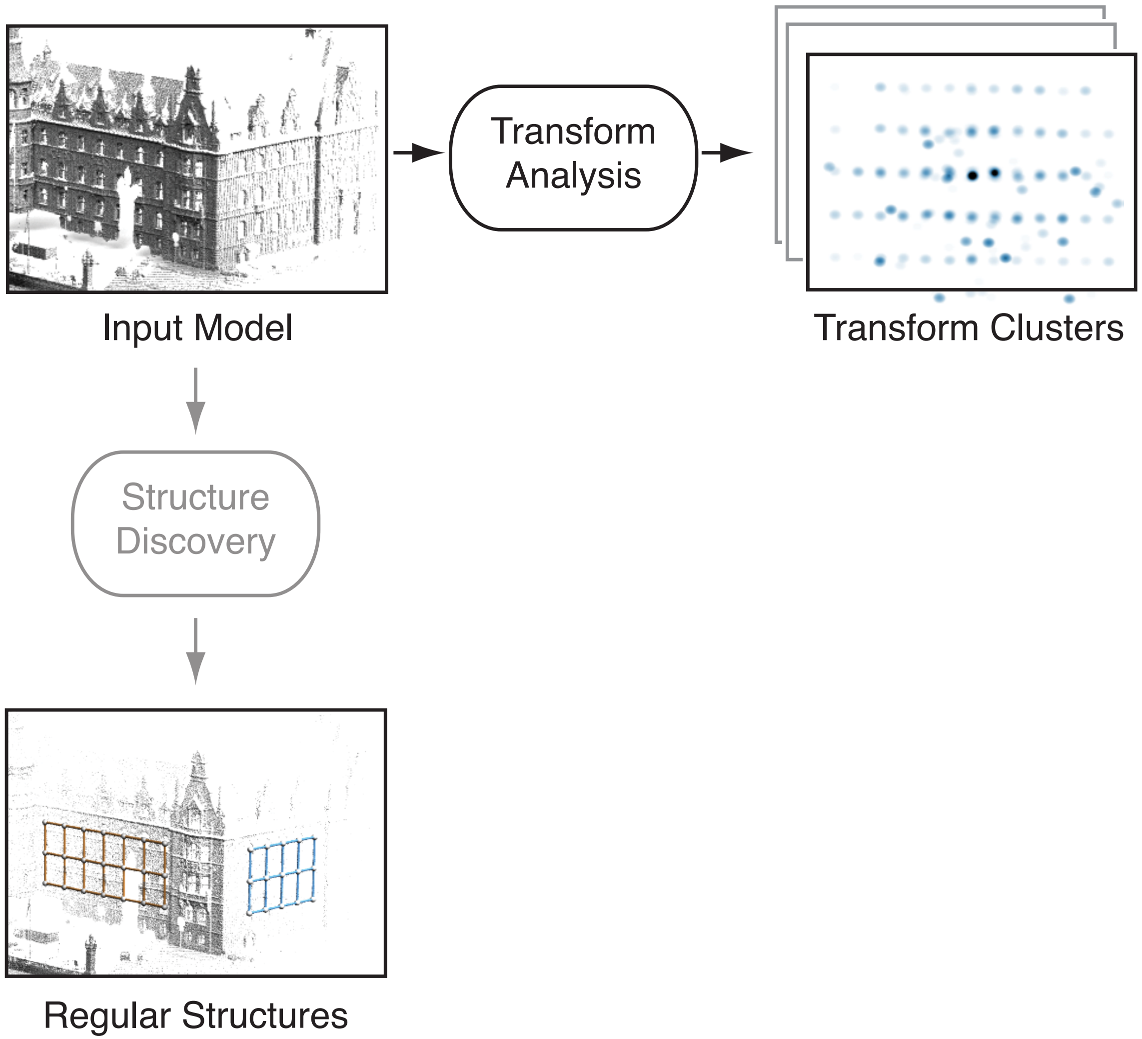
[Pauly et al. 2008]

Structure Discovery

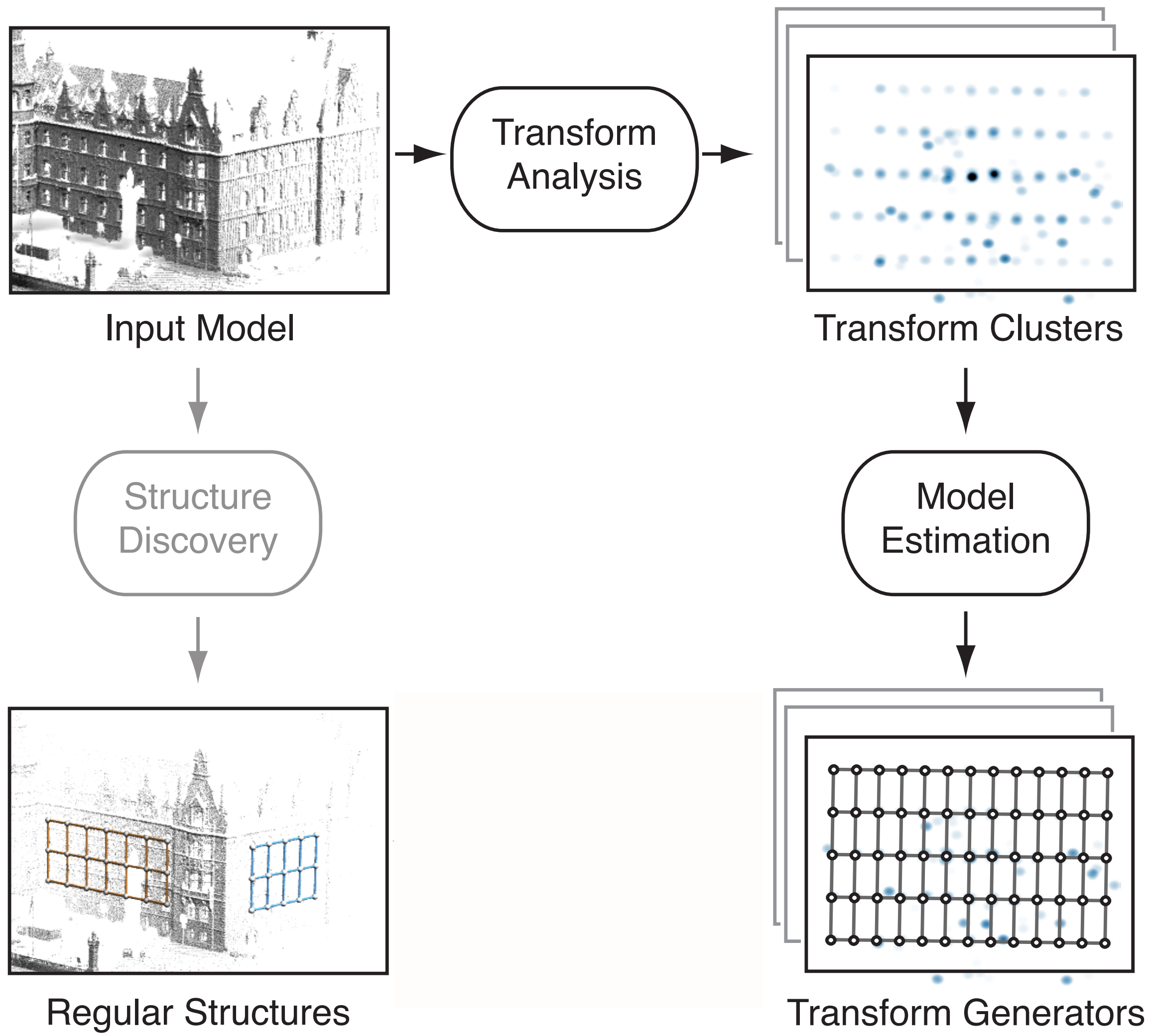


Input Model

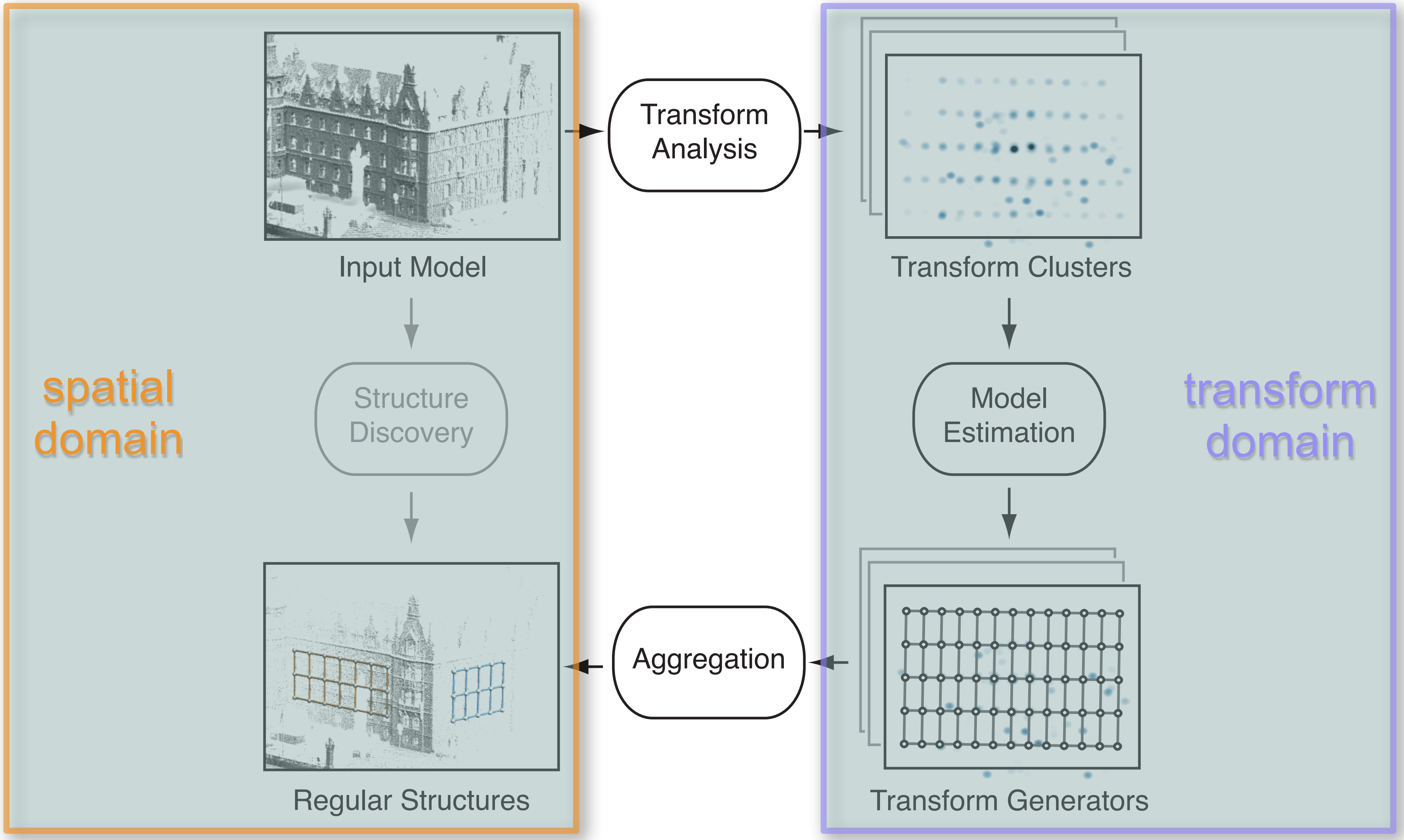
Structure Discovery



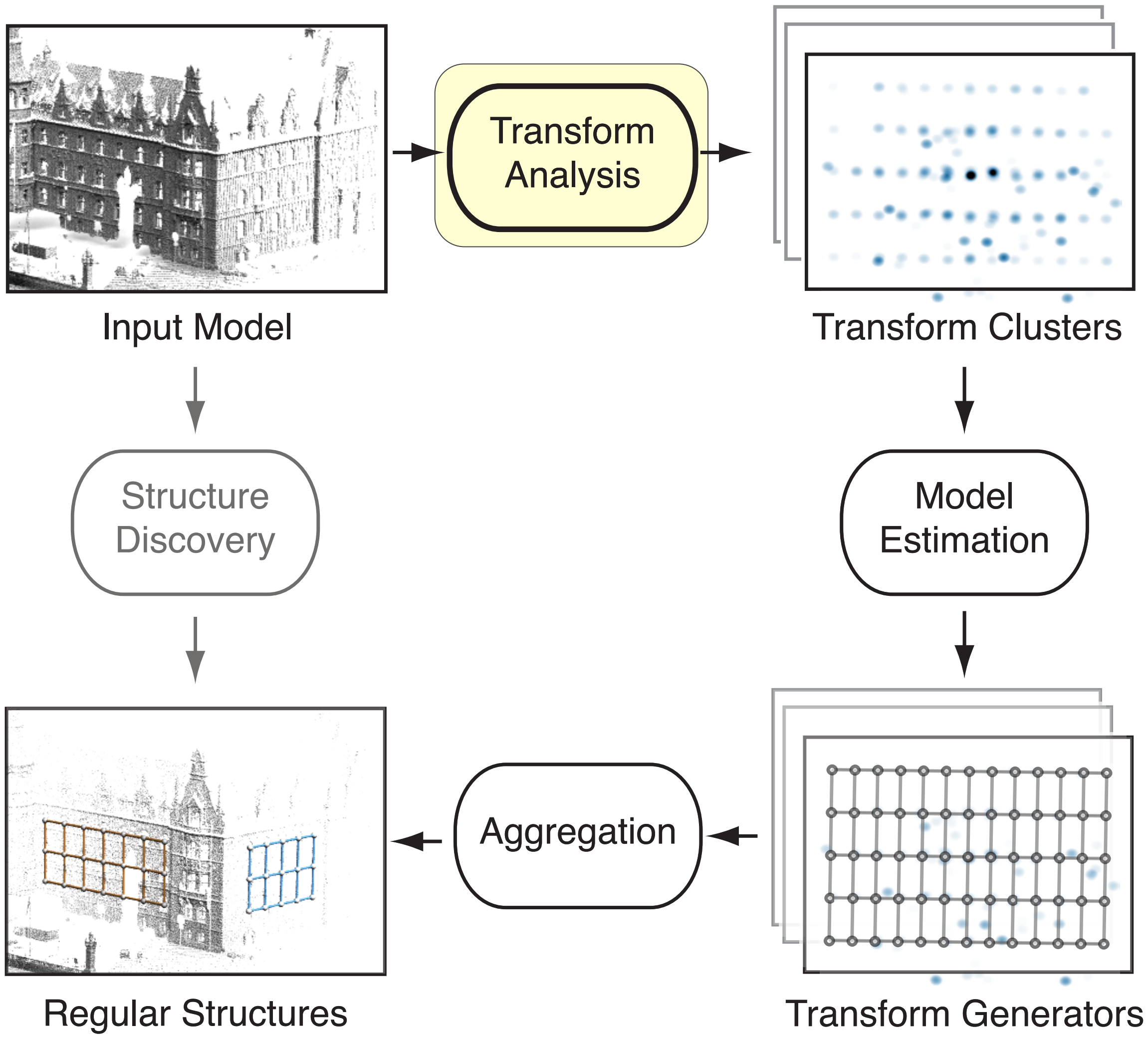
Structure Discovery



Structure Discovery

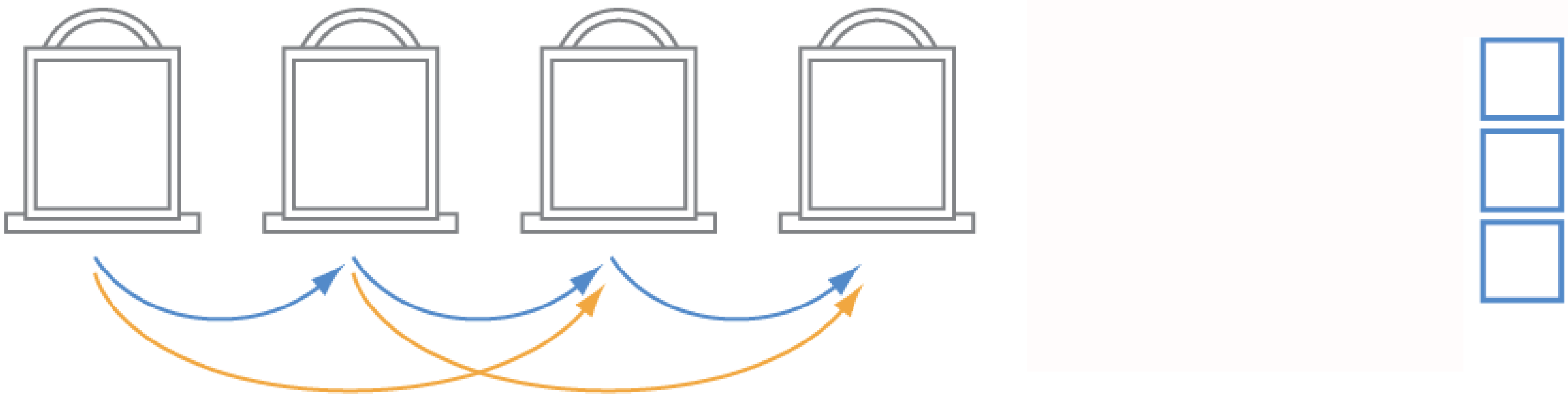


Structure Discovery

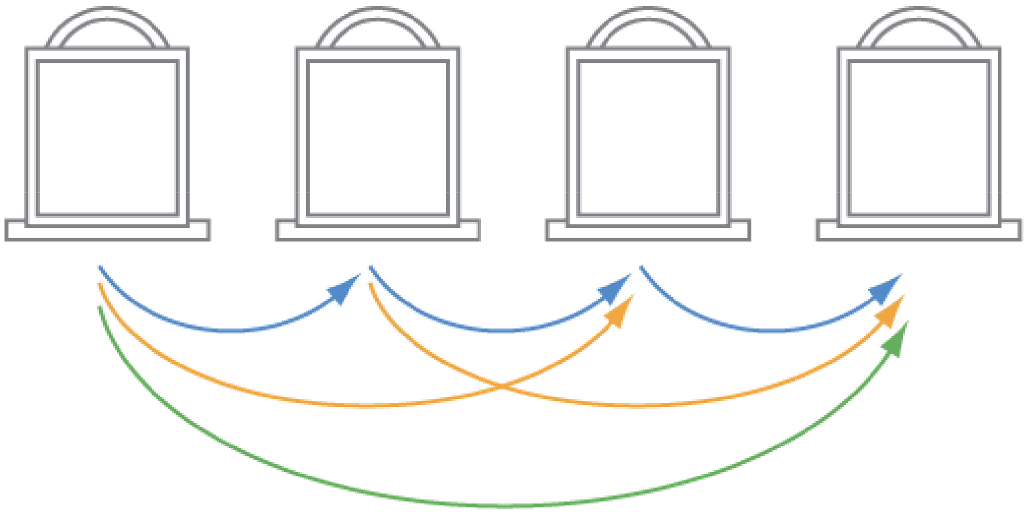


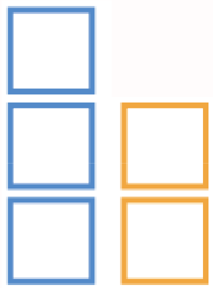
Transformations

Transformations

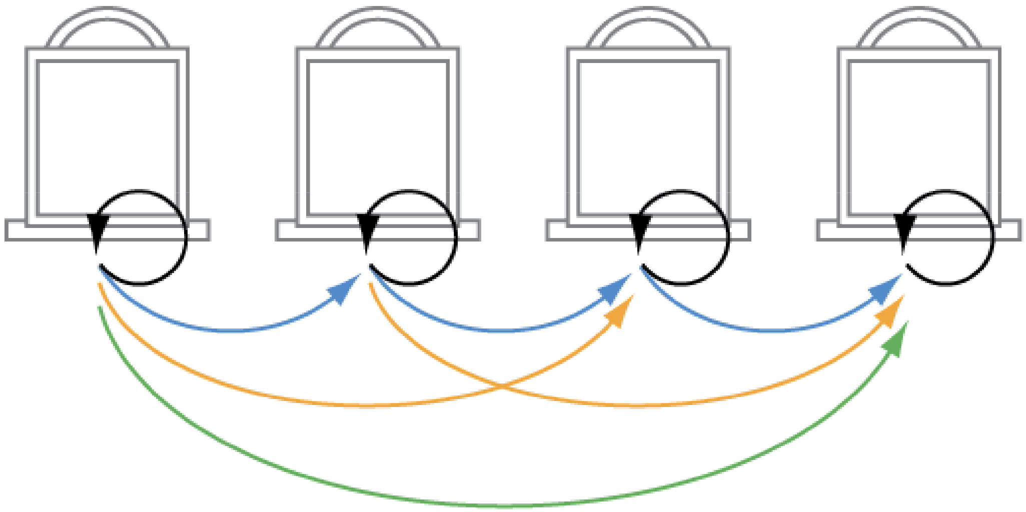


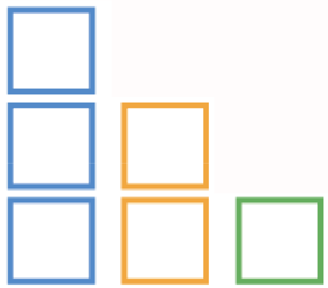
Transformations



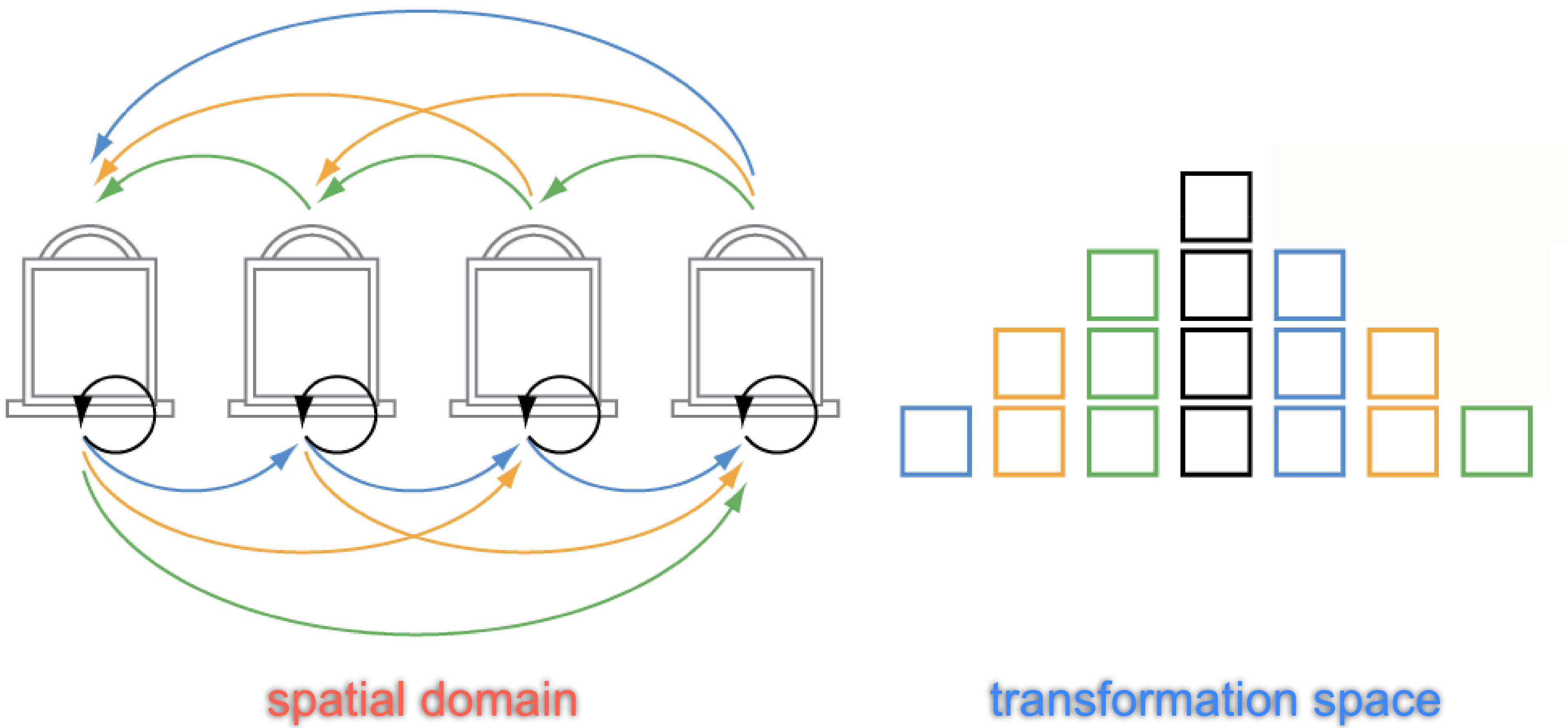


Transformations

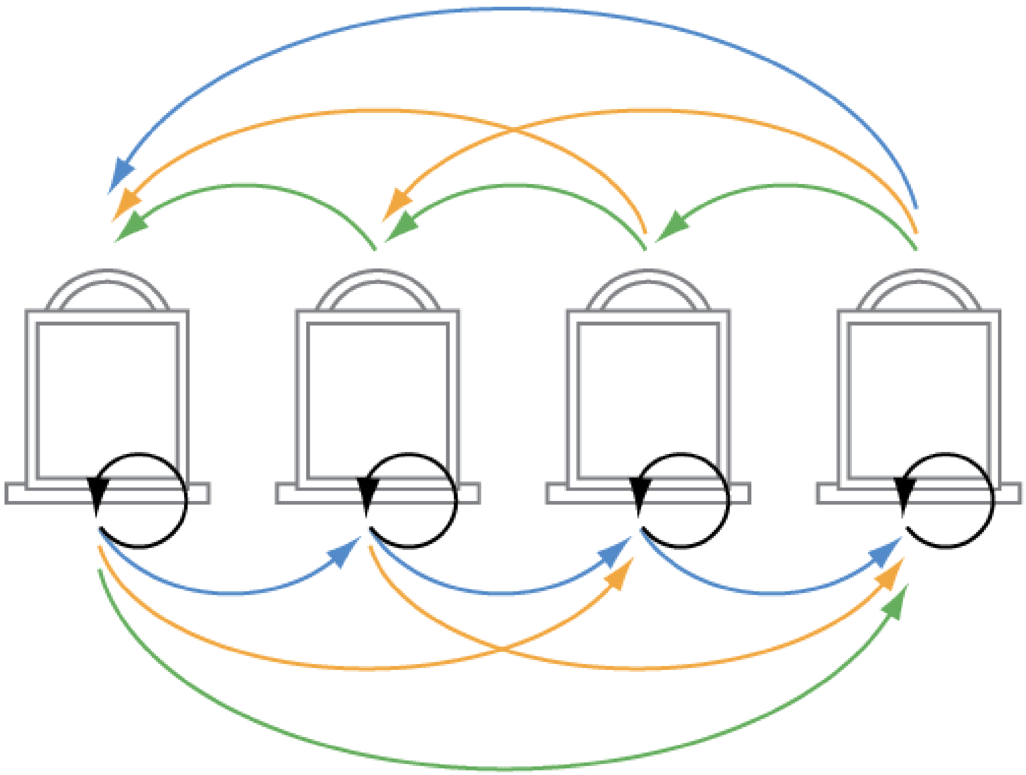




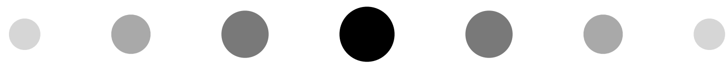
Transformations



Transformations

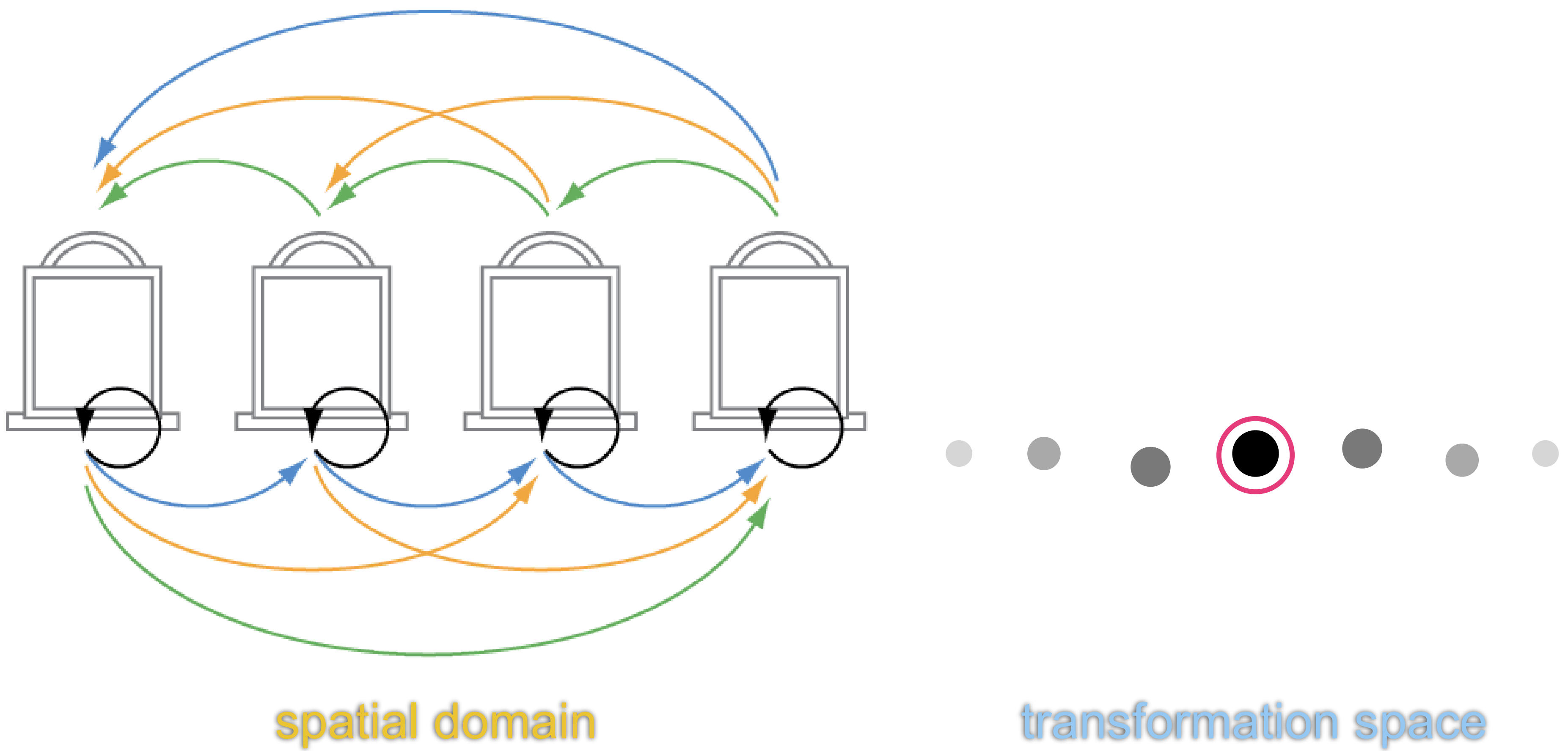


spatial domain

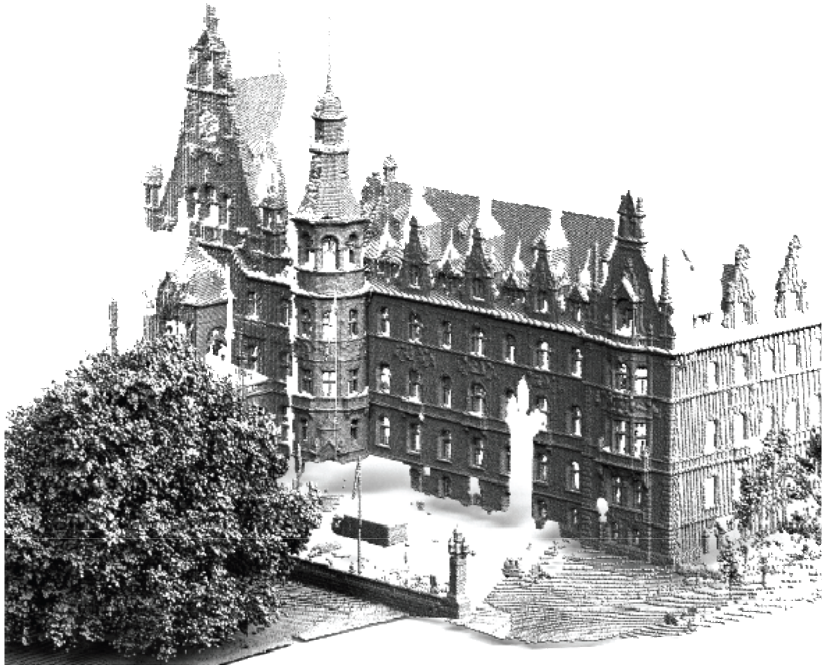


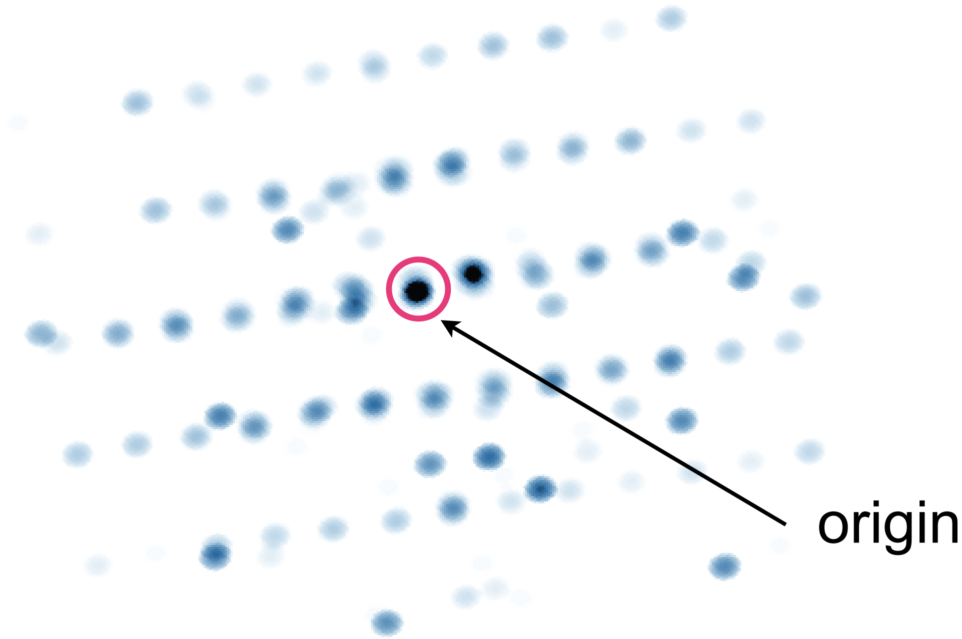
transformation space

Transformations



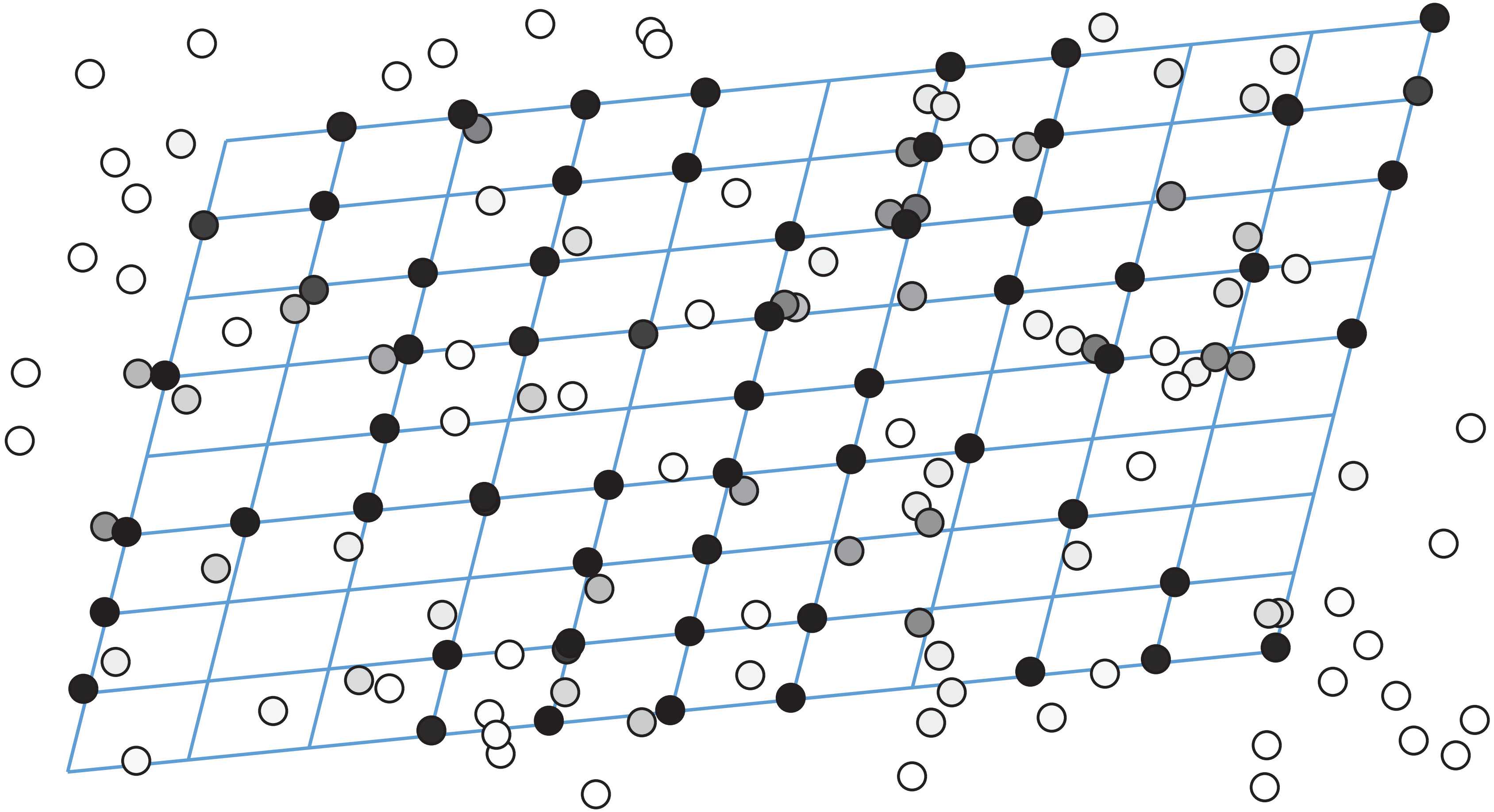
Model Estimation



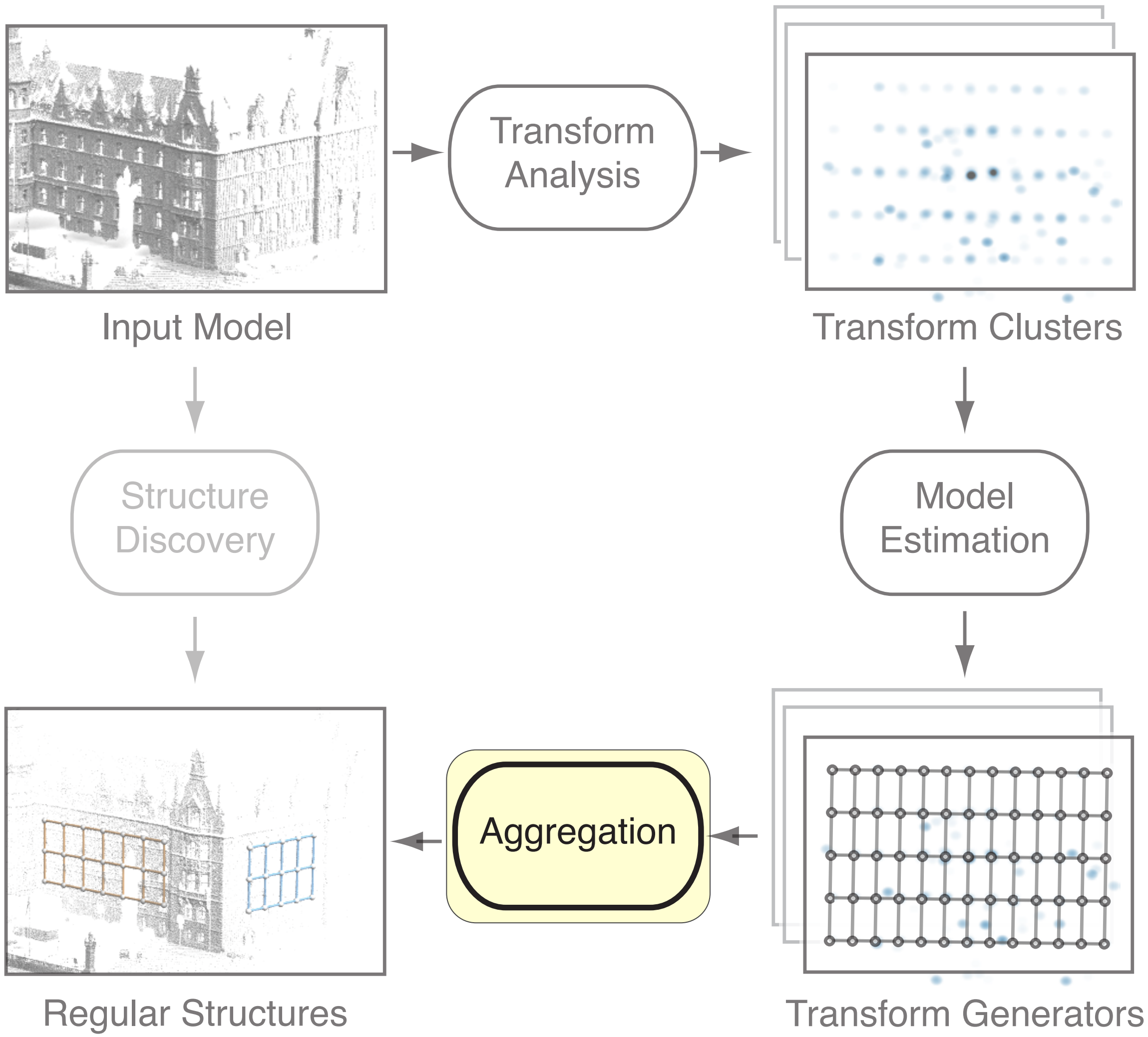


density plot of
pair-wise transformations

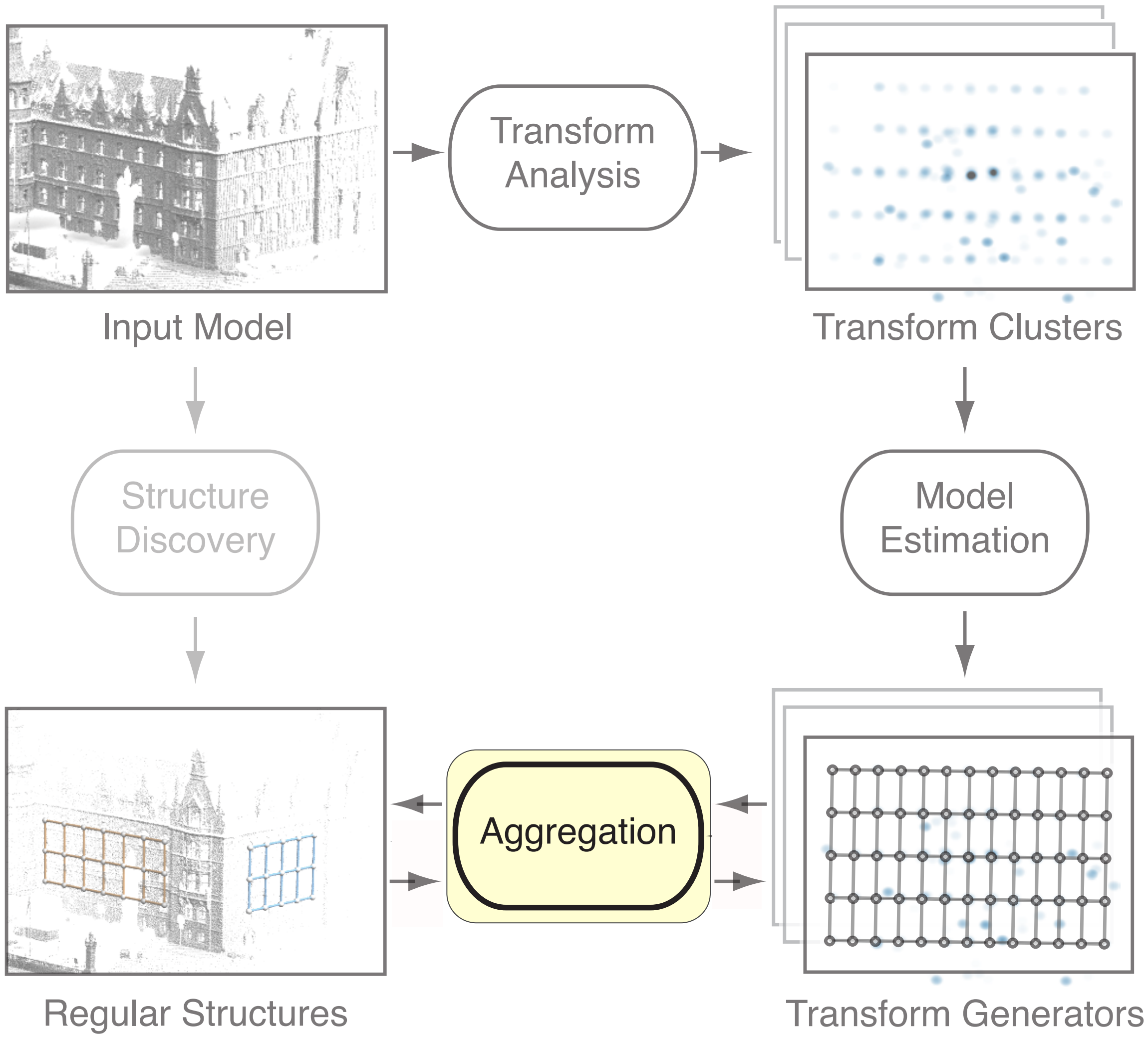
Optimization in Transform Domain



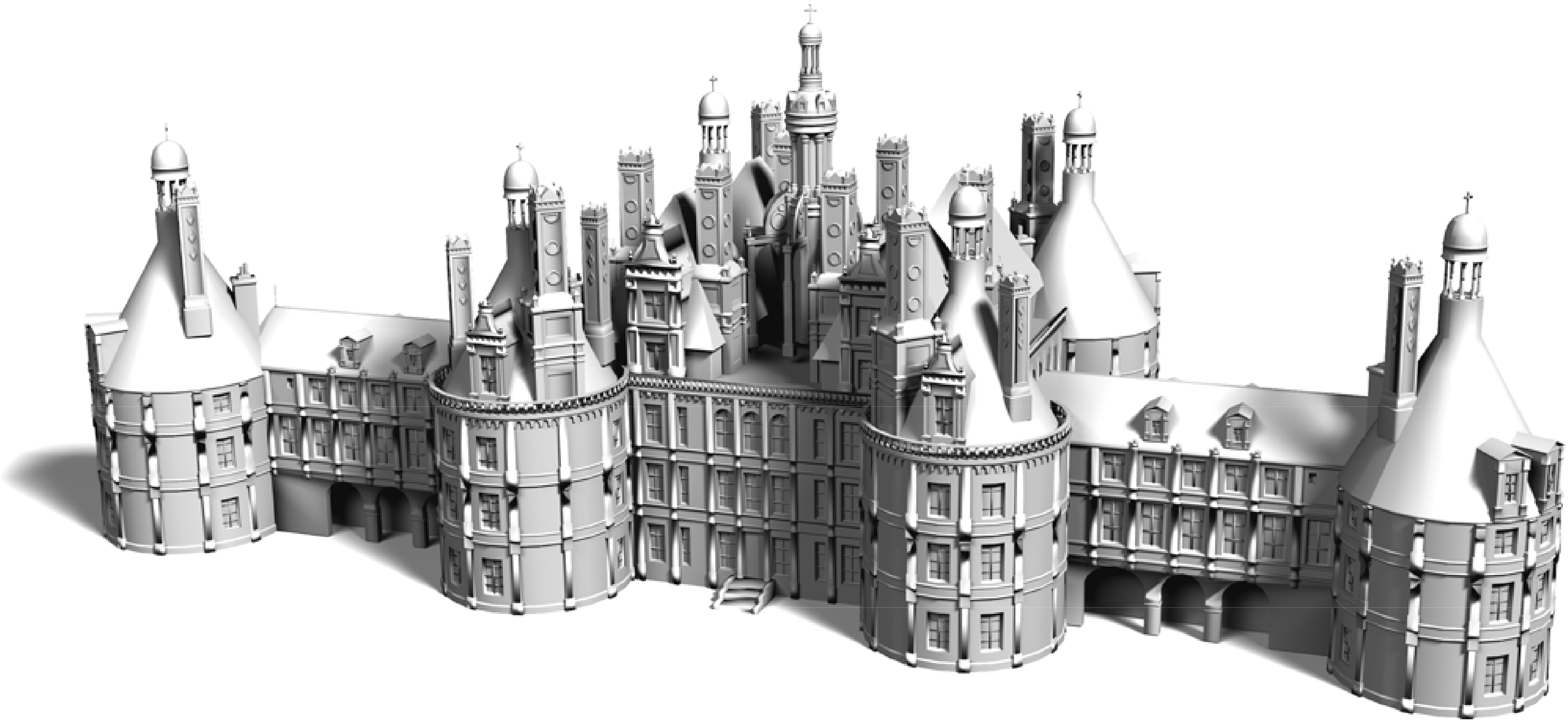
Structure Discovery



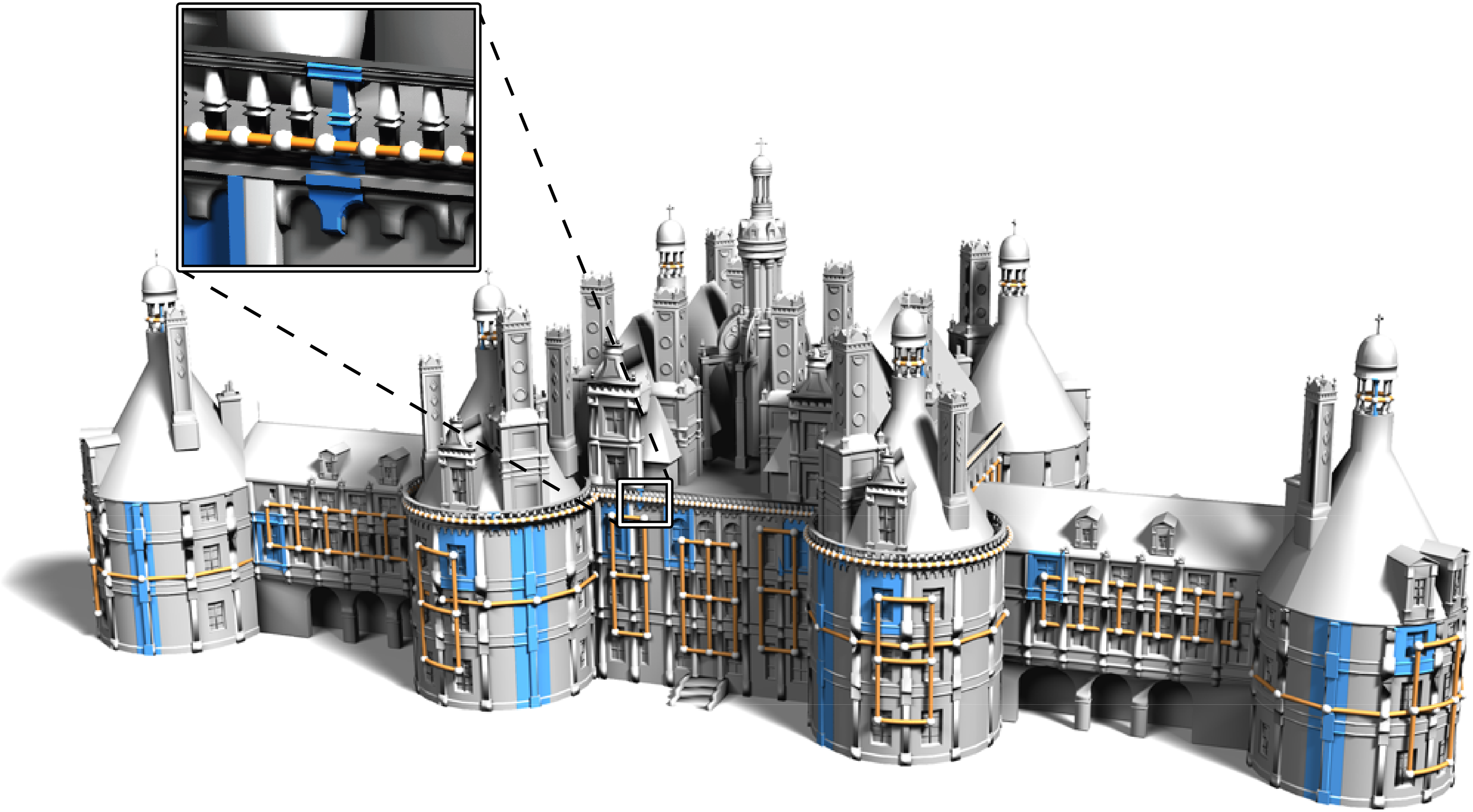
Structure Discovery



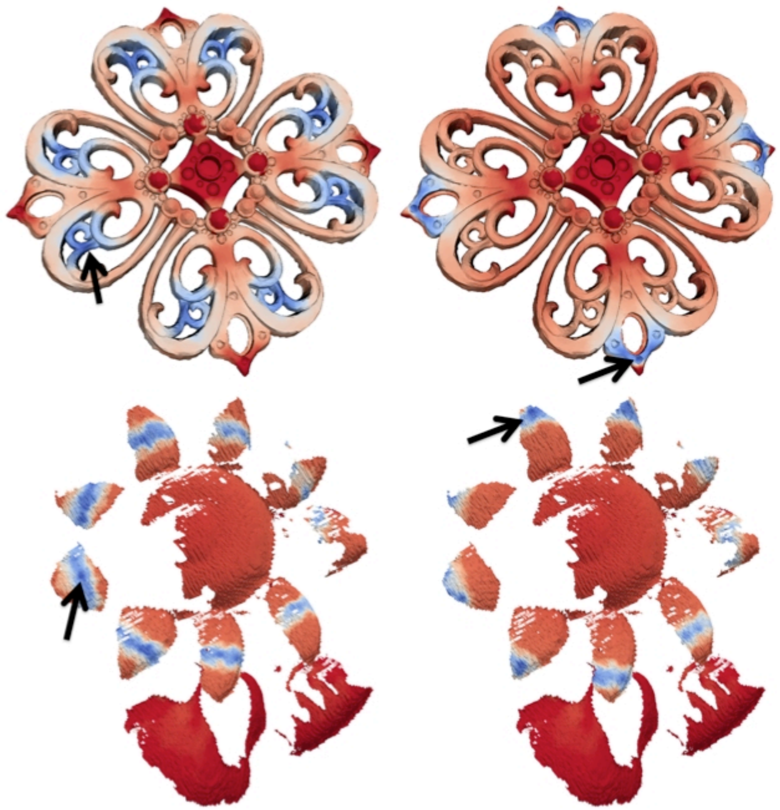
Chambord Castle

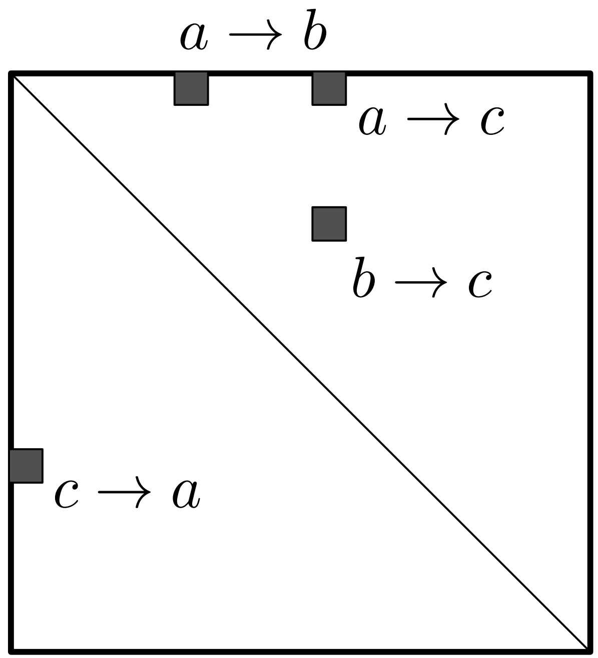


Chambord Castle



Symmetry Factored Encoding





finding cliques *amounts to* spectral analysis

[Lipman et al. 2009]