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Non-conforming surface representations

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Abstract

Surface geometry is commonly represented by a collection of primitives. Conforming representations consist of primitives meeting at their boundaries (e.g., in a triangle mesh two triangles are incident upon an edge). Without the restriction to conforming elements there are no dependencies among primitives, leading to more degrees of freedom for each primitive. This yields more efficient and flexible algorithms for reconstruction, processing, and rendering, as well as compact and accurate representations.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Line and Curve Generation

1. Extended Abstract

Shapes are represented in computer graphics mostly by collections of primitives. Typically, primitives are connected, yielding at least a C^0 approximation of the surface. Lately, surface representations based on disconnected primitives are becoming more and more popular.

Piecewise approximations of curves and surfaces could be classified based on the complexity of each piece and the degree of continuity along the boundaries of pieces. A canonical example are piecewise polynomial surfaces. The complexity of pieces equates with the polynomial order. Usually, higher polynomial order is exploited to maximize the degree of continuity among patches.

Following this reasoning, piecewise linear curves are continuous but have discontinuous tangents (i.e. C^0 continuous curve); piecewise quadratic curves are tangent continuous but have discontinuous curvatures (C^1), piecewise cubic curves have continuous curvatures (C^2), and so on. A point based representation would complement this set of representations at the lower end of degrees and continuities. Based on the correspondence of polynomial degrees and continuities it could be viewed as C^{-1} continuous.

Points are interesting because they naturally result from most acquisition systems and are could be used as a rendering primitive [ZPvG01]. By connecting a surface to the set of points [Lev03, AA03, AK04] points can also be used for modeling shapes [ZPKG02].

However, one could well deviate from the idea of using the degrees of freedom per segment (i.e. the polynomial degree) to maximize the continuity of the compound representation and rather optimize other measures as the error to the original shape. The promise is that more degrees of freedom lead to more accurate shape approximations. The idea of representations with individually varying degrees of freedom per segment and continuity among segments is illustrated in Figure 1. In particular, we can show that a set of linear surface patches (i.e. polygons) can approximate a given surface with smaller *symmetric Hausdorff error* than connected linear pieces.

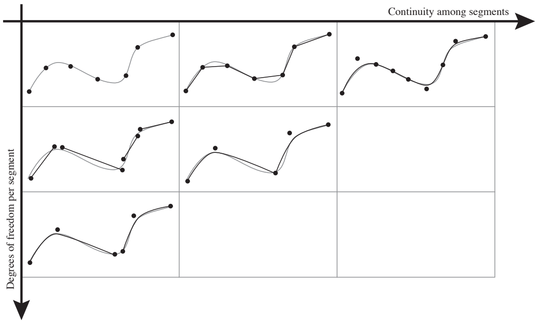

Figure 1: *Representations of a curve with polynomial pieces of varying order and different continuity at segment boundaries.*

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Note that most shape approximation approaches only optimize one sided approximation errors, i.e. the distance of each point on the surface being approximated to the closest point on the approximation. A cardinal example is the reconstruction of a surface from points, where mostly the distance of the input points to the reconstructed surface is optimized; yet, parts of the reconstructed surface might be arbitrarily far away from the point set.

Figure 2 illustrates the problem of one-sided distances and how disconnected primitives can improve the Hausdorff error: A unit circle $\mathcal{C}$ is approximated by three line segments $\mathcal{L}$ – connected in the upper row (i.e., a triangle) and disconnected in the lower row. In the first column vertices of the triangle are placed on the circle, which is the common approach for computing PL approximations. The distance is symmetric and we have $d(\mathcal{C},\mathcal{L}) = d(\mathcal{L},\mathcal{C}) = 1/2$. Disconnecting this set of lines and changing the length cannot improve this distance. If line segments are chosen to be tangent to the circle, we find that the distance from circle to triangle $d(\mathcal{C},\mathcal{L}) = 1/2$ but the distance from triangle to circle is $(\mathcal{L},\mathcal{C}) = 1$. Disconnecting the line segments and optimizing their length leads to an overall distance $h(\mathcal{C},\mathcal{L}) = 1/2$. Optimizing the line segments for a symmetric error yields the results shown in the right column. The triangle has $h(\mathcal{C},\mathcal{L}) = 1/3$ but for the line segments we find $h(\mathcal{C},\mathcal{L}) = 1/4$.

The other main advantage of using disconnected pieces is computational: Each piece can be approximated independently from others. This results in algorithms that can process large sampled surfaces with only a small memory overhead.

The basic idea for computing sets of disconnected shape representations is to use a spatial partitioning and to approximate the surface individually in each cell. Common types of cells are boxes resulting from spatial subdivisions (i.e. an octrees or kd-tree) [OBA*03], or spheres resulting from a bounding hierarchy [OBS04].

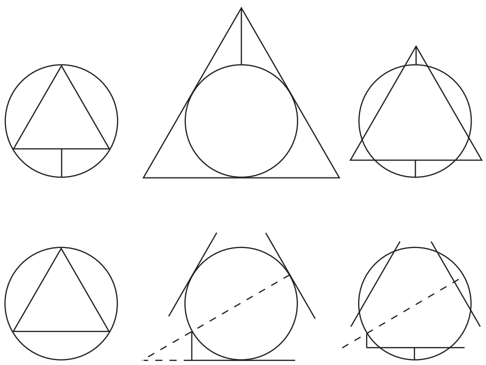

Figure 2: *Approximation of a circle by three connected (upper row) or disconnected (lower row) line segments.*

If the individual pieces have very small error they could be rendered directly [ABCO*01]. However, commonly the individual would leave visible gaps. Several better options for rendering are:

- If the pieces are described in implicit form and sufficiently overlapping, they could be blended and the resulting zero set is a smooth surface. [OBA*03, OBS04].
- The set could be converted to a closed surface using recent reconstruction approaches [Lev03, SOS04].
- Similarly to recent splatting approaches [BSK04] the pieces could be scan converted and blended in images space.

The last approach leads to fast and high quality rendering of the representation. In addition, it also provides smooth normals and curvatures by blending these quantities in screen space as well.

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