

# A study of the Manipulability of the PHANToM™ OMNI™ Haptic Interface

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## Abstract

When a haptic interface is going to be used as a part of a computational system a design criteria consist of maximizing the coincidence between the application 3D space and the physical volume where the haptic device provides the maximum performance. A well known parameter to evaluate the performance of a mechanical manipulator is the manipulability. This paper explains in detail the analysis of manipulability of the Phantom Omni haptic interface including the study of the manipulability distribution into its real workspace boundaries.

Categories and Subject Descriptors (according to ACM CCS): I.3.4 [Computer Graphics]: Graphics Utilities: Virtual Device Interfaces H.5.2 [User Interfaces]: Haptic I/O, Input Devices and Strategies

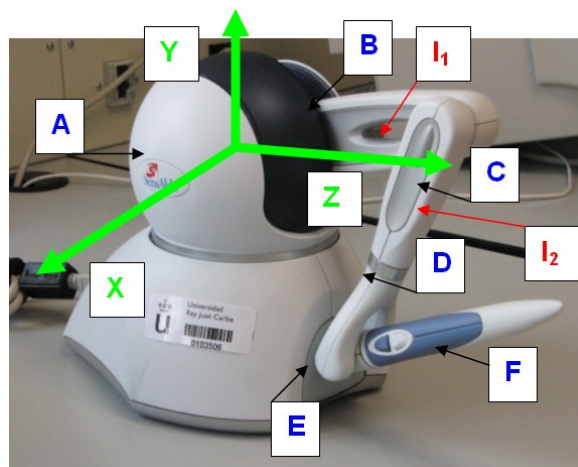
## 1. Introduction

One of the characteristics that define the performance of a haptic device is the manipulability  $\mu$ . A definition is that manipulability is the skill for transmit movement and to apply forces in arbitrary directions [PK98]. In this paper we perform an analysis of the PHANToM OMNI manipulability. The result is a map of curves of iso-manipulability that then is faced with the application workspace. We can consider three different definitions of workspace:

- Nominal Workspace (Fig. 1): It is the volume in which the manufacturer guarantees the specified force feedback and precision. For the OMNI device it is a rectangular prism of dimensions 160 W x 120 H x 70 D mm.,
- Real Workspace: It is the volume that the End Effector can reach.
- Effective Workspace: It is the volume in which the OMNI will actuate according to the application.

## 2. Forward Kinematics

The first step is to solve the forward kinematics problem:  $(x, y, z) = F(\theta_1, \theta_2, \theta_3)$ . Having in account the geometrical relations between the elements in Fig. 1, we obtain the trans-



**Figure 1:** Different Components of the OMNI device. Coordinate System (CS) XYZ in the origin. Arms  $l_1 = 129$  mm and  $l_2 = 133$  mm.

formation matrix from End Effector regard to the CSO:

$$\begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \sin \theta_3 & \cos \theta_3 \sin \theta_1 & l_1 \cos \theta_2 \sin \theta_1 + l_2 \sin \theta_1 \sin \theta_3 \\ 0 & \cos \theta_3 & \sin \theta_3 & -l_2 \cos \theta_3 + l_1 \sin \theta_2 \\ -\sin \theta_1 & -\cos \theta_1 \sin \theta_3 & \cos \theta_1 \cos \theta_3 & l_1 \cos \theta_1 \cos \theta_2 + l_2 \cos \theta_1 \sin \theta_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

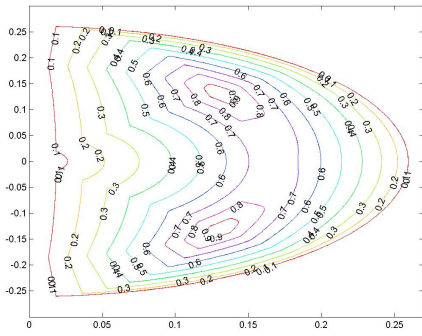


Figure 2: Isomanipulability curves map for  $X = 0$  plane.

Then, the coordinates of the End Effector are:  $x = (l_1 \cos \theta_2 + l_2 \sin \theta_3) \sin \theta_1$ ,  $y = (l_1 \sin \theta_2 - l_2 \cos \theta_3)$ ,  $z = (l_1 \cos \theta_2 + l_2 \sin \theta_3) \cos \theta_1$ .

### 3. Inverse Kinematics

The second step is to solve  $(\theta_1, \theta_2, \theta_3) = I(x, y, z)$ . Again it is obtained from the OMNI geometrical characteristics:  $\theta_1 = -\arctan(\frac{y}{z})$ ;  $\theta_2 = -\arctan(\frac{y}{H}) + \arccos(\frac{L^2 + l_1^2 - l_2^2}{2l_1L})$  with  $(H^2 = x^2 + z^2)$ ;  $\theta_3 = \arctan(\frac{H - l_1 \cos \theta_2}{l_1 \sin \theta_2 - y})$

### 4. Jacobian

The third step is to solve the differential model (Jacobian Matrix J) establishing the relations between angular velocities of the joints and those of the End Effector of the device.

$$J = \begin{pmatrix} l_1 \cos \theta_2 + l_2 \sin \theta_3 & 0 & 0 \\ 0 & l_1 \cos(\theta_2 - \theta_3) & 0 \\ 0 & -l_1 \sin(\theta_2 - \theta_3) & l_2 \\ 0 & 0 & -1 \\ \cos \theta_3 & 0 & 0 \\ \sin \theta_3 & 0 & 0 \end{pmatrix}$$

### 5. Manipulability

The formulation of the manipulability value is [CFT02, TPM04] defined like  $\mu = \sigma_{\min}(J_u) / \sigma_{\max}(J_u)$ , where  $\sigma_{\min}$  and  $\sigma_{\max}$  are the minimum and maximum singular values of the matrix  $J_u$ , upper half of Jacobian matrix. Figure 2 shows a map of curves of iso-manipulability for the OMNI when  $\theta_1 = 0$ .

### 6. Real Manipulability

Now we obtain the section of the map of manipulability that corresponds with the real workspace of the OMNI. This is

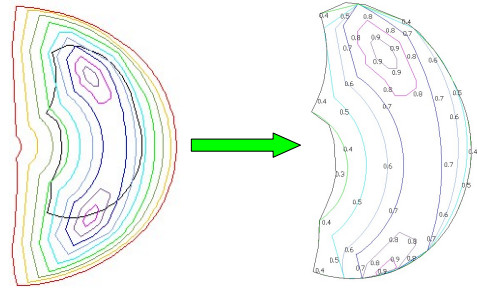


Figure 3: Projection of the real workspace on the manipulability map and Subspace of manipulability defined for the real workspace.

the maximum area that End Effector can reach. The resultant intersection (Fig. 3) shows, that the chosen area has the best values of manipulability. We have found an optimum manipulability zone of the OMNI device at inter-arms angle values of  $l_1$  and  $l_2$  near to  $90^\circ$ , coinciding with the central area of curves in Fig. 3, with an optimum value in the upper zone of the map.

### 7. Conclusions

It has been calculated the area where it will be desirable that the Effective Workspace is located. By selecting the location of the OMNI properly we will improve the performance of the manipulator, increasing efficiency of its transmission of velocity and torque to the force feedback point End Effector.

### Acknowledgments

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