Diffusion Tensor Weighted Harmonic Fields for Feature Classification

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Abstract

This paper presents an efficient method for feature definition and classification on shapes. We tackle this challenge by exploring the weighted harmonic field (WHF), which is also the stable state of a heat diffusion regulated by an anisotropic diffusion tensor. The technical merit of our method is highlighted by the elegant integration of locally-defined diffusion tensor and globally-defined harmonic field in an anisotropic manner. At the computational front, the partial differential equation of heat diffusion becomes a linear system with Dirichlet boundary condition at heat sources (also called seeds). We develop an algorithm for automatic seed selection, enhanced by a fast update procedure in a high dimensional space. Various experiments are conducted to demonstrate the ease of manipulation and high performance of our method.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Curve, surface, solid, and object representations

1. Introduction

Feature definition and classification have been of great practical importance in many graphics tasks and applications. Extensive studies on feature extraction, while continuing for more than ten years, have been gaining momentum because multi-type features can assist registration, segmentation, shape analysis and understanding, and many more. Features can be classified into multiple types that may include point feature, curve feature, patch feature, etc. [KCL09]. The fundamental goal of this paper is to advocate an integrated strategy for feature identification and region clustering, and develop a robust and efficient method to classify multi-type features of curved geometry.

Multi-type (e.g., point, curve, and patch) features offer much more general and convenient tools for shape analysis than point features alone. Defining multi-type features is extremely challenging due to the diversity of features scattered across arbitrarily curved geometry. Most existing work is focusing on point features, while some recent research starts to address curve features [SJW*11] by connecting points, or multi-type features by using tensor voting [KCL09]. The theory of tensor voting has demonstrated great advantages in modeling tasks such as clustering and feature recognition [MTL00, PSK*02, LDB05, KCL09]. Since the voting tensor is only a local quantity, it is very sensitive to noise, resulting in degraded performance when being employed to distinguish weak features from noise. Due to the lack of knowledge for global shape information, tensor voting alone falls short in finding large-scale patch features. This paper

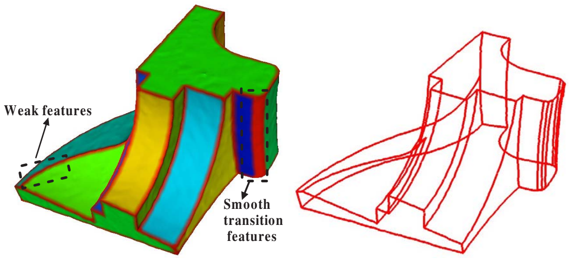


Figure 1: *Multi-type feature classification on the Fandisk with Gaussian noise ($\sigma = 5\%$ of mean edge length). Patch features are shown in different colors, while curve and point features are colored in red. Our method can detect weak features and smooth transition features (left). The connected curve features are highlighted as a wire-frame representation (right).*

DOI: 10.2312/PE/PG/PG2011short/093-098

serves for this urgent need. Our novel idea lies at the integrated strategy that aims to unite a global diffusion process with a local tensor-voting method. Fig. 1 highlights multi-type features extracted by our method.

Diffusion process, which is intrinsically related to the probability of random walk [LHMR09, ZZC11], is a powerful toolkit in combating noise. Recent years have witnessed a great accomplishment in the rapid development of heat diffusion and relevant algorithms on manifolds [DMSB99, SOG09, SCV10]. It elegantly bridges the large gap between local and global geometry via time scale. Recent work oftentimes concentrates on dynamic solutions of the partial differential equation (PDE) of heat diffusion, which typically requires eigenfunctions of the Laplace-Beltrami operator and convolutions of heat kernels. When the diffusion arrives at its final stable state (i.e., a harmonic field), the PDE degenerates to a linear system. A direct usage of the diffusion process will naturally give rise to the smooth transition among nearby regions without having evident clues on various feature types and their meaningful classification. Furthermore, most previous heat diffusions are isotropic. In essence, anisotropic diffusion, which is much more powerful than the standard isotropic diffusion, can control the diffusion direction by assigning weighted diffusion operators spatially to different regions.

In this paper, we explore the definition and classification of multi-type features based on diffusion tensor weighted harmonic fields, which collectively inherit the advantages of local geometric tensors and global harmonic fields. The local geometric tensor we formulated is a versatile diffusion tensor, which can well control the anisotropic diffusion, and properly distinguish weak features from noise, cluster curve features by using their principal diffusion directions, etc. Fig. 2 illustrates the pipeline of our approach. We use normal voting tensor and diffusion tensor to obtain a primary classification. After calculating the WHF, we further classify the vertices into different type features by considering the values of the high dimension harmonic field which will be introduced in Section 2.4. Note that our seeds are automatically selected during the iterative classification procedure which will also be introduced in Section 2.4.

2. Feature classification based on WHFs

Given a triangular mesh, our method classifies all the vertices into features with different types. Our new method is founded upon the computation of anisotropic harmonic fields, and comprises four steps: diffusion tensor calculation, initial feature analysis, numerical construction of the anisotropic diffusion process, and feature classification.

2.1. Diffusion tensor

The normal voting tensor $\mathbf{T}(v_i)$ of a vertex v_i can be computed by the sum of the weighted covariance matrices [MTL00, PSK*02],

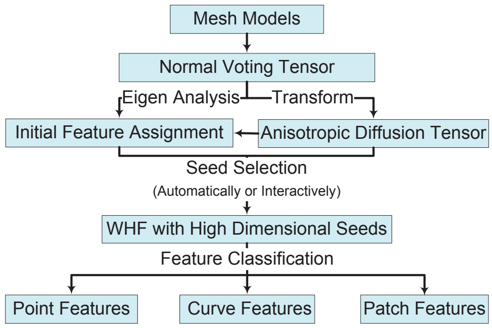


Figure 2: *The functional pipeline of our new method. We use normal voting tensor to distinguish weak features from noise, and design anisotropic diffusion to determine different types of features.*

$$\mathbf{T}(v_i) = \sum_{t_j \in N_t(v_i)} \mu_j \mathbf{n}_{t_j} \mathbf{n}_{t_j}^T, \tag{1}$$

where t_j is a triangle, $N_t(v_i)$ denotes the set of neighboring triangles of v_i, $\mathbf{n}_{t_j}$ is the normal of triangle t_j, and μ_j is the weight. To accommodate meshes with long and narrow triangles, we modify the weight μ_j as

$$\mu_j = \frac{area(t_j)}{area_{max}} exp\Big(-\frac{\|c_j - v_i\|}{\|c_j - v_i\|_{max}}\Big), \tag{2}$$

where $area(t_j)$ is the area of triangle t_j, $area_{max}$ is the maximum area among $N_t(v_i)$, c_j is the barycenter of triangle t_j, and $\|c_j - v_i\|_{max}$ is the maximum value among the neighboring triangles of v_i.

Since the normal voting tensor is a positive semi-definite tensor with second order, it can be diagonalized by eigenvalues ($\lambda_1 > \lambda_2 > \lambda_3 \geq 0$) and reformulated by a spectral representation

$$\mathbf{T}(v_i) = \lambda_1 \mathbf{e}_1 \mathbf{e}_1^T + \lambda_2 \mathbf{e}_2 \mathbf{e}_2^T + \lambda_3 \mathbf{e}_3 \mathbf{e}_3^T, \tag{3}$$

where $\mathbf{e}_k$ is the corresponding eigenvector of λ_k. The three eigenvectors of a normal voting tensor are orthogonal, and the eigenvalues characterize the diffusion velocities along the corresponding directions. Directly adopting the normal voting tensor as the diffusion tensor will lead to rapid diffusion when cutting across the sharp edges and slow diffusion when traveling along them. This is opposite to our goal, thus, we construct our anisotropic diffusion tensor as

$$\mathbf{D}(v_i) = \widetilde{\lambda}_1 \mathbf{e}_1 \mathbf{e}_1^T + \widetilde{\lambda}_2 \mathbf{e}_2 \mathbf{e}_2^T + \widetilde{\lambda}_3 \mathbf{e}_3 \mathbf{e}_3^T, \tag{4}$$

where $\widetilde{\lambda}_i = exp(-\frac{\lambda_i}{\delta_D})$, $i = 1, 2, 3$, with diffusion parameter δ_D that controls the diffusion velocities. According to the theory of Rayleigh quotient [HJ85], the diffusion velocity from the vertex v_i along a vector $\mathbf{e}$ can be expressed as $vel(v_i, \mathbf{e}) = \frac{\mathbf{e}^T \mathbf{D}(v_i) \mathbf{e}}{\mathbf{e}^T \mathbf{e}}$. It can be viewed as the length of the vector projected onto the ellipsoid.

Usually, the principal diffusion direction is the most informative one, which is defined as the direction corresponding to the maximal diffusion velocity. The principal diffusion directions are also used to distinguish weak features and noise, and guide the feature curve growing and merging.

2.2. Initial feature assignment

According to the eigen-analysis [KCL09] and the neighboring relationship, we assign vertices to different initial types : face, strong-edge, weak-edge, corner, and noise. Since the relative difference between eigenvalues is crucial for classifying vertices, the eigenvalues are normalized to bring consistency into different data, using $\frac{\lambda_i}{\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}$, $i = 1,2,3$. For convenience, all the eigenvalues mentioned later are normalized, and we still denote as λ_i.

The types of corner, face, and strong-edge can be easily determined by the corresponding eigenvalues. For each vertex, if ($\lambda_3 > 0.1$), mark the vertex corner vertex; if ($\lambda_2 < 0.02$), mark the vertex face vertex; if ($\lambda_2 > 0.1$ & $\lambda_3 < 0.02$), mark the vertex strong-edge vertex. To separate weak-edge vertices and noise vertices, we design a criterion, called *neighboring vertex coincidence* (NVC), by considering more neighbors. It utilizes the phenomenon that the neighboring vertices of a noise vertex usually have different principal diffusion directions. Specifically, given a vertex v_i not belonging to face vertices, we put it into a front set. Along its principal diffusion direction, we find non-face vertices in its neighbors, which have similar principal diffusion directions (intersecting angle less than 15°). If such coincident vertex exists, we mark it as the new front and keep this front tracking procedure going. If the number of found coincident vertices is larger than 2, we call vertex v_i satisfies the NVC criterion. Then the weak-feature vertex is identified, if ($0.02 \leq \lambda_2 \leq 0.1$ & $\lambda_3 < 0.02$ & NVC). The rest vertices that are not marked are noise vertices. Note that these parameters are chosen through our experiments.

After the initial feature assignment, we mark corner vertices as point features, and add noise vertices into the set of face vertices, which will be further classified into patch features or curve features. The weak-edge vertices will be enhanced through merging, and then connected into the set of strong-edge vertices, which are considered as the constituents of curve features. The separate process of weak-edge vertices is helpful to distinguish weak curve features from noise. Since the diffusion is a global PDE that has a built-in resistance to noise, a simple enhancement procedure by way of merging suffices for our goal. We can enhance the weak-edge vertices by simply enlarging the second eigenvalue of the voting tensor $\lambda_2 = \alpha\lambda_2$, where α is a parameter that is set as α=10. As a result, the gap between features and noise are magnified, which is necessary for classification purpose.

2.3. Weighted harmonic fields

It is well known that the heat diffusion over a manifold M is governed by the heat equation. We formulate the weighted diffusion process as

$$\begin{cases} \frac{\partial u(v,t)}{\partial t} = -div(\widetilde{\mathbf{D}}\nabla u(v,t)), & t \in R^+, \\ u(v,t) = c(v), & v \in S, \\ u(v,0) = 0, & v \in others, \end{cases} \tag{5}$$

where the diffusivity $\widetilde{\mathbf{D}}$ is a 3×3 symmetric matrix, S is the set of seeds, and $c(v)$ is the fixed value of seed v. The weight matrix $\widetilde{\mathbf{D}}$ serves for two purposes: encoding diffusion tensor $\mathbf{D}$, and characterizing geometric difference between neighboring vertices. These local attributes are crucial for our feature classification, which will be addressed next.

From the entire model's perspective, we allow heat propagation to reach its global equilibrium and consider the stable stage of the weighted diffusion in Eq. (5). When the diffusion has reached its final stable state, time t is omitted in the notation. Then, Eq. (5) reduces to

$$\begin{cases} div(\widetilde{\mathbf{D}}\nabla u(v)) = 0, & \\ u(v) = c(v), & v \in S, \end{cases} \tag{6}$$

whose solution is a WHF. The discrete formulation of Eq. (6) can be written into matrix form $\mathbf{LF} = 0$, subject to Dirichlet boundary condition $\mathbf{F}(v) = c(v), v \in S$, where $\mathbf{L}$ is the $n \times n$ Laplace matrix, and $\mathbf{F}$ is the harmonic field. The Laplace matrix $\mathbf{L}$ has elements $\mathbf{L}_{ij} = -K(v_i, v_j)$, $\mathbf{L}_{ii} = \sum_{j \in N(i)} K(v_i, v_j)$, with the kernel given by $K(v_i, v_j) = exp(-\frac{(v_i - v_j)^T (w_{ij}(\mathbf{D}(v_i)+\mathbf{D}(v_j))^{-1}(v_i - v_j)}{\delta_k})$, where δ_k is a control parameter and is set to the inverse of the maximal eigenvalues of diffusion matrices $\mathbf{D}(v_i)$ and $\mathbf{D}(v_j)$. The weight is defined as $w_{ij} = exp(-\frac{\|\mathbf{NCC}_i - \mathbf{NCC}_j\|}{\delta_G})$, where $\mathbf{NCC}_i$ denotes normal-controlled coordinates [WHSQ11] of vertex v_i and δ_G is a parameter. In such a way, we can very well characterize the geometric differences of neighboring vertices. Since the symmetric matrix $\mathbf{D}(\cdot)$ is positive definite and $w_{ij} = w_{ji}$ is a positive constant, the value of $K(\cdot,\cdot)$ is within interval (0,1]. Moreover, it is easy to obtain that $K(v_i, v_j) = K(v_j, v_i)$, which satisfies the symmetry of heat diffusion.

The above-documented formulation is a linear system with boundary condition at seeds. For large meshes, it is necessary to avoid solving the linear system every time after we update seeds. We adopt the popular *Penalty method* [XZCOX09] to fast update the seeds. Specifically, $\mathbf{L}$ is symmetric which admits fast Cholesky factorization and fast updating of Cholesky [DH09]. As a result, adding/removing seed constraints can be written as matrix additions. On the other hand, we want to utilize the harmonic field for clustering. We let each harmonic field associate with one seed valued as 1, and the other seeds valued as 0. The region where the values are most similar to the current seed in the harmonic field is treated as a patch feature. Hence, we put

Algorithm 1: Classification

for *i=1:n* **do**
  find $[f_{ij}, j] = \max_j(F_{ij})$;
  if *v_i is a non-face vertex* || $f_{ij} < 0.5$ **then**
    continue;
  end
  add the vertex v_i to the j-th patch feature;
  assign it the color value j;
  $f_i = \sum_j F_{ij}$;
  if *f_i is the local extremum in the direction perpendicular to its principal diffusion direction (the common boundary of two patch features)* **then**
    mark v_i as a vertex of curve features;
  end
end

seeds in a high dimensional space R^d, where d is the number of seeds. Now $\mathbf{F}$ becomes a $n \times d$ unknown matrix, which represent d harmonic fields. Then, Eq. (6) can be rewritten as

$$(\mathbf{L} + \bar{\mathbf{P}})\mathbf{F} = \bar{\mathbf{P}}\mathbf{B}, \bar{\mathbf{P}} = \mathbf{P} + \mathbf{U}\mathbf{U}^T - \mathbf{D}\mathbf{D}^T, \tag{7}$$

where the $n \times n$ penalty matrix $\mathbf{P}$, the $n \times n$ modification matrices $\mathbf{U}$ and $\mathbf{D}$, and the $n \times d$ constraint matrix $\mathbf{B}$ have the following entries:

$$\mathbf{P}_{ij} = \begin{cases} \alpha & i = j \in C, \\ 0 & \text{otherwise}, \end{cases} \quad \mathbf{U}_{ij} = \begin{cases} \sqrt{\alpha} & i = j \in C_{ins}, \\ 0 & \text{otherwise}, \end{cases}$$

$$\mathbf{D}_{ij} = \begin{cases} \sqrt{\alpha} & i = j \in C_{del}, \\ 0 & \text{otherwise}, \end{cases} \quad \mathbf{B}_{ij} = \begin{cases} 1 & i \text{ is the } j\text{th in } \bar{C}, \\ 0 & \text{otherwise}, \end{cases}$$

with α being the penalty factor, C the indices for the previous seeds, C_{ins} the indices for newly-inserted seeds, C_{del} the indices of seeds to be deleted, and $\bar{C}$ the indices for the updated seeds. As for the penalty factor, we choose $\alpha = 1.0 \times 10^8$ for all the examples in our current implementation. It may be noted that, the penalty method only handles soft constraints, so α value must be large enough to confine the values at seeds within the desirable range.

2.4. Feature classification

With the harmonic fields obtained from the diffusion PDE in Eq. (7), one can easily classify the vertices by their values of each element in R^d. Since in the initial feature assignment, most types of the vertices have already been classified except for the face vertices, we only need to handle the face vertices V_f, and classify them into different patch features or curve features via Algorithm 1. The main idea is to cluster the face vertices with the most similar field values into a patch feature.

We devise a scheme to automatically select the seeds for the feature classification (documented in Algorithm 2). The seeds are randomly selected from the unclassified face vertices, during the iterative procedure. In principle, since our method is solely based on the global diffusion and its anisotropic nature, it is insensitive to the exact location of seeds. Note that, the number of seeds is the number of patch features. The proposed method is efficient, since the updating of Cholesky factorization is very fast.

Algorithm 2: Automatic seed selection

input : Cholesky factorization of the Laplacian matrix $\mathbf{L}$, face vertices V_F.
output: Classification result.
Initialize: $C = \emptyset$, $\bar{C} = \emptyset$, $C_{del} = \emptyset$.
while *V_F is not empty* **do**
  1: set $C_{ins} = \emptyset$, randomly select a seed from V_F, and add it to C_{ins} and $\bar{C}$.
  2: fast update the Cholesky via Eq. (7), and obtain the updated $\mathbf{F}$.
  3: classify the face vertices using Algorithm 1, and set C=$\bar{C}$.
end

After this step, the vertices have already been classified into different features. For curve features useful in downstream graphics applications, it generally requires a concise representation by ordering the vertices and forming a link among them. In our work, this can be easily handled using the principal diffusion directions. Smooth feature curves are found by edge-tracking along the principal diffusion directions. For an edge of M, if both endpoints belong to point features or curve features, and at least one of the principal diffusion directions is close to the edge (intersecting angle less than 15°), the two endpoints are connected to form a line segment contributing to a feature curve. Our merging method is easy and effective, with no post-processing needed.

3. Experimental results and discussions

In this section, we demonstrate the performance of our method by conducting various experiments. All the experiments shown below are conducted on a computer with 1.6GHz Intel Core (TM, 4Core/8Threads) i7 CPU with 4G RAM, where both synthetic and scanned meshes are utilized. Most computational tasks of our method can be carried out in the pre-processing stage, such as the diffusion tensor, initial feature assignment, and Cholesky decomposition of the Laplace matrix.

Fig. 1 and Fig. 3 show the feature classification on noisy meshes. We add Gaussian noise with $\sigma = 5\%$ of mean edge length to the input data. Since both the local diffusion tensors and the global harmonic fields are utilized, our method inherits the robustness to noise from the diffusion.

In Fig. 4, we compare our method with two other related methods: the random walk (RW) [LHMR09] (b), and the

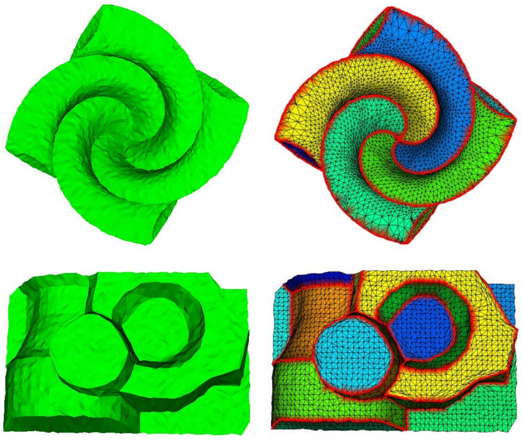
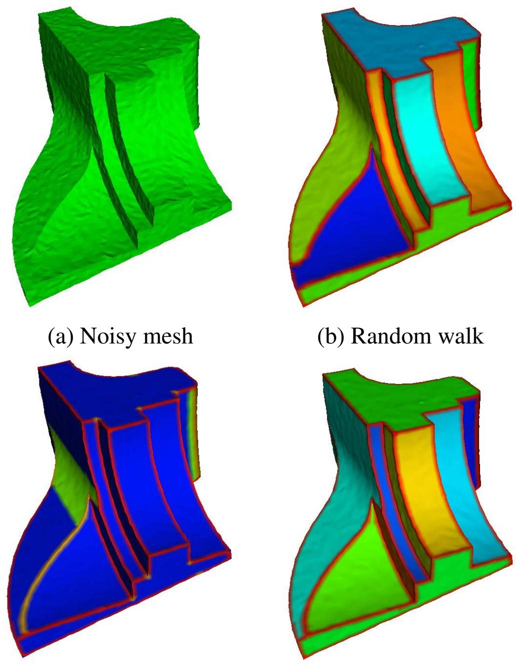


Figure 3: *Feature classification on noisy meshes: The Octa-flower (top) and the Stamp (bottom).*

Table 1: *Time (second) comparison of different methods.*

Data (# V)	RW	TV	Ours
Fandisk (6477)	0.44s	3.54s	0.15s
Tre-twist (12800)	1.78s	5.13s	0.29s
Pawn (68000)	8.64s	24.16s	1.24s
Canstick (170000)	38.25s	86.47s	5.58s



(a) Noisy mesh (b) Random walk

(c) Tensor voting (d) Our method

Figure 4: *Comparison of different methods on the Fandisk with noise.*

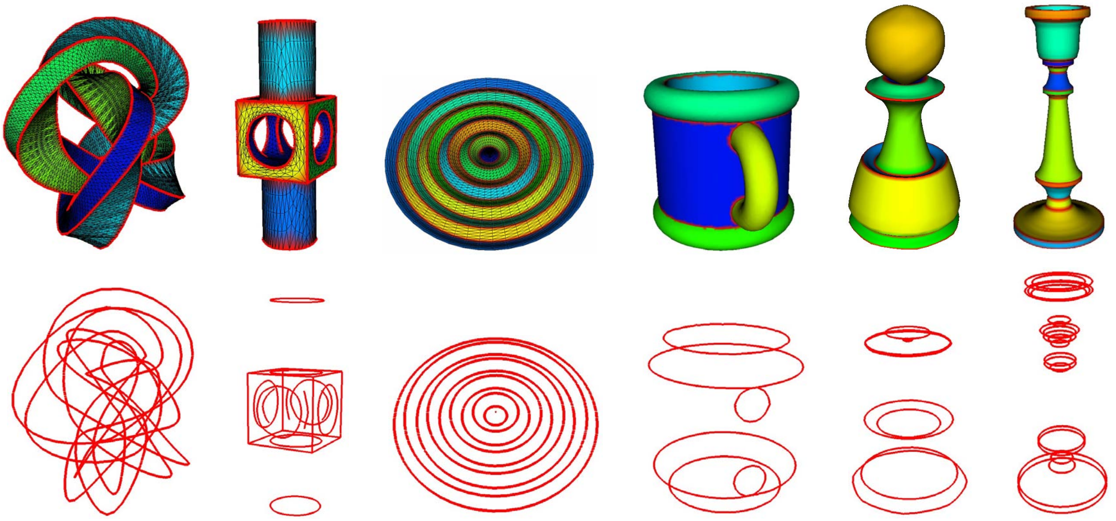
tensor voting (TV) [KCL09] (c). The RW method fails to find weak features, and it also has difficulties in clustering vertices along a curve feature. Moreover, since the matrix of a random walk is not symmetric, fast Cholesky factorization is not applicable, which greatly reduces its computational speed. The TV method can detect weak features and smooth transition regions, but the clustering is sensitive to noise. Consequently, much post-processing effort is unavoidable. Besides, neither can distinguish different patch features. Instead, they consider the entirety collectively as one single patch. Our method is able to classify the vertices into different types of features, with both weak features and smooth transition features detected on noisy meshes (Fig. 1 and Fig. 4 (d)). Table 1 summarizes the running time of these algorithms. Note that the pre-processing time is not listed for all three methods. Our method performs better than those methods in terms of both speed and clustering results. More experimental results are shown in Fig. 5 to further demonstrate the performance of our method. The connected curve features are highlighted on the bottom row.

Certain limitations may still exist, which call for further improvement. From the feature's perspective, meshes without closed curve features do not have separated patch features in principle. This is because the heat diffusion can not be easily stopped when propagating across the connected regions. The proposed method, which aims at feature classification, does not further segment the connected patches. Such challenge is further compounded by model uncertainty and noise, especially for generally near smooth models. To address this challenge, one would require separate segmentation algorithms for further classification, such as multi-objective segmentation [SNKS09], which is one of our future research direction.

4. Conclusion and future work

In this paper, We have articulated a new method for multi-type feature classification on meshes, based on diffusion tensor weighted harmonic fields. A diffusion tensor has been locally designed to collectively control the global anisotropic behavior of heat diffusion. Such diffusion tensor has also been utilized for forming feature curves along its principal diffusion directions. The novelty of our method centers at the elegant integration of locally-defined diffusion tensor and globally-defined harmonic field in an anisotropic manner. The proposed algorithm is capable of rapidly updating the underlying harmonic fields with real-time performance. Our feature classification method is also insensitive to seed selection and robust to noisy meshes.

For immediate future work, we are planning to extend our method to handle diverse types of geometric and scientific data, such as point clouds in urban architecture modeling, volumetric data in medical imaging, and higher dimensional



(a) Tre-twist (b) Block (c) Cosine plane (d) Cup (e) Pawn (f) Canstick

Figure 5: *More examples. Top: Feature classification results. Bottom: The connected curve features.*

manifolds in scientific disciplines. Moreover, applying this approach to vector field design and non-photorealistic visualization deserves further investigation, which can broaden our method's application scopes.

Acknowledgments

This research is supported in part by the Fundamental Research Fund for the Central Universities, National Natural Science Foundation of China-Guangdong Joint Fund grant U0935004, National Natural Science Foundation of China grant 60873181, and US NSF grants IIS-0710819, IIS-0949467, IIS-1047715, and IIS-1049448. Models are courtesy of the AIM@SHAPE repository.

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